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# Multivariable systems design: a two ships controller for replenishment at sea.

Lima, Celso Graca

# MULTIVARIABLE SYSTEMS DESIGN: A TWO SHIPS CONTROLLER FOR REPLENISHMENT AT SEA

Celso Graça Lima

# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

MULTIVARIABLE SYSTEMS DESIGN:
A TWO SHIPS CONTROLLER
FOR REPLENISHMENT AT SEA

by

Celso Graça Lima

June 1974

Thesis Advisor: George Thaler

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The mathematical model of the ships motion in three degrees of freedom is established and implemented for the formation of digital computer programs and approximated for the study of steady state decoupling.

Using parameter optimization techniques, the necessary control loops are designed with the aid of a digital computer. The station keeping problem of underway operation is simulated and the results compared and analyzed.

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Multivariable Systems Design:
A Two Ships Controller
For Replenishment at Sea

by

Celso Graça Lima
Lieutenant Commander, Brazilian Navy
Brazilian Naval Academy, 1959
B.S., Naval Postgraduate School, 1973

Submitted in partial fulfillment of the
requirements for the degree of

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from the

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# ABSTRACT

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# I. INTRODUCTION

## A. REPLENISHMENT AT SEA: TWO SHIPS SEEN AS A MULTIVARIABLE SYSTEM

The operational procedure of replenishing ships at sea while steaming on parallel courses in close proximity came into general use during World War II and it is still used by the Navy, for the purpose of safe transferring of the maximum amount of cargo in a minimum of time, in order to enable them to operate at sea for prolonged periods.

As the cargo must be guided and controlled during the transfer operation, a suitable physical connection must be established and maintained between the two ships as they travel along with identical speeds. This connection requires that the ships operate at close quarters, a fact that makes the maneuver critical and dangerous.

In this Thesis a special type of approach and the important phase of maintaining station will be considered.

Steaming alongside results in certain hydrodynamic phenomena that create not completely understood interaction forces and moments between the ships, that generate the ever existing danger of collision.

Investigation of manual and automatic control of two individual ships in a replenishment at sea operation has been carried out using digital and hybrid computer simulation [2, 5, 12]. The present work introduces a different way of analyzing the problem. The two ships are seen as a multivariable system, their dynamics being coupled by the interaction effects observed when steaming at close proximity.

This section includes a summary of experimental results about the interaction effects; it is followed by the derivation of the equations of motion for one ship under calm water conditions and the extension for two

ships, establishing the multiple input, multiple output (MIMO) model. An analysis of steady state decoupling is performed and a compensator that makes the system have a desired transient response is designed with the aid of a digital computer, using parameter optimization techniques.

## B. THE INTERACTION EFFECTS

When underway there are areas of increased water pressure at the bow and stern of a ship, and decreased pressure (suction) amid ships as the result of differences in velocity of the flow of the water around the hull. When the ships are alongside each other underway, this venturi effect is increased and becomes further complicated because of the intermingling of the pressure areas of the two ships. Changes in relative position between ships will impose rapid changes in the pressure effects on their hulls.

It is therefore evident that to maintain station while alongside, a certain amount of rudder is usually necessary [4]. It will depend on the size and load of both ships, sea and wind conditions, speed and separation. As a result of increased rudder, speed is reduced, which complicates the station keeping problem, because it increases the handling difficulties of the ships, and it is also dangerous if a rudder casualty should occur.

The classic and original work on the reaction of vessels underway and in close proximity to one another was the investigation carried out by Taylor [14]. Further theoretical and experimental studies [11, 13] shown agreement as far as the major trends are concerned.

It has been proved that the navigational risks are greater during the process of taking up or breaking away from the abeam position. There will be situations when both the interaction forces and moments tend to draw one ship toward the other; the rudder angles should be such that the

rudder moments oppose the interact moments, but the simultaneous rudder forces would tend to add to the force of attraction. Therefore the rudder must be deflected sufficiently so that not only the interaction moment is overcome, but also a yaw angle is introduced that creates an outboard force that counteracts both the attraction and rudder forces. By these means the ships should be able to avoid collision, provided there is enough transverse separation between them, so that the available rudder forces can effectively correct the inward swing.

The two ships have to apply opposite rudder to keep on parallel courses. In the approach phase, the rudder has to swing from a relatively large deflection to the other side. The precise timing of this command is not easily chosen but the maneuver will be correctly executed with a proper automatic controller.

## II. SHIP'S EQUATIONS OF MOTION

### A. DERIVATION FOR THE S.I.S.O. CASE

Bodies moving in a fluid medium are free to move in six degrees of freedom. In order to define the equations of motion, a right hand rectangular coordinate system is established, the origin of which is chosen to be in the body itself, as shown in Figure II-1. The origin and the axis are fixed with respect to the body but movable with respect to another system of coordinate axis fixed in space; it is assumed that at the initial time of the problem the two systems coincide.

The motion of a rigid body is expressed by Newton's Laws of Motion:

$$\text{(external forces)} \quad \vec{F}_e = \frac{d}{dt} \text{(momentum)}$$

$$\text{(external moments)} \quad \vec{M}_e = \frac{d}{dt} \text{(angular momentum)} \qquad \text{(II-1)}$$

The equations describing the ship's six degrees of freedom have been found [1] to be:

$$
\begin{aligned}
X &= m\left[\dot{U} - RV + QW - x_G(R^2+Q^2) + y_G(PQ-\dot{R}) + z_G(PR+\dot{Q})\right] \\
Y &= m\left[\dot{V} - PW + RU + x_G(\dot{R}+PW) - y_G(P^2+R^2) + z_G(PQ-\dot{P})\right] \\
Z &= m\left[\dot{W} - QU + PV + x_G(PR-\dot{Q}) + y_G(\dot{P}+QR) - z_G(Q^2+P^2)\right] \\
L &= \dot{P}I_x + (I_z - I_y)QR + m\left[y_G(\dot{W}-QU+PV) - z_G(\dot{V}-PW+RU)\right] \\
M &= \dot{Q}I_y + (I_x - I_z)PR + m\left[z_G(\dot{U}-RV+QW) - x_G(\dot{W}-QU+PV)\right] \\
N &= \dot{R}I_z + (I_y - I_x)PQ + m\left[x_G(\dot{V}-PW+RU) - y_G(\dot{U}-RV+QW)\right]
\end{aligned}
\qquad \text{(II-2)}
$$

Satisfying the following equations

$$
\begin{aligned}
\vec{F}_e &= \vec{i}X + \vec{j}Y + \vec{k}Z \\
\vec{M}_e &= \vec{i}L + \vec{j}M + \vec{k}N
\end{aligned}
$$

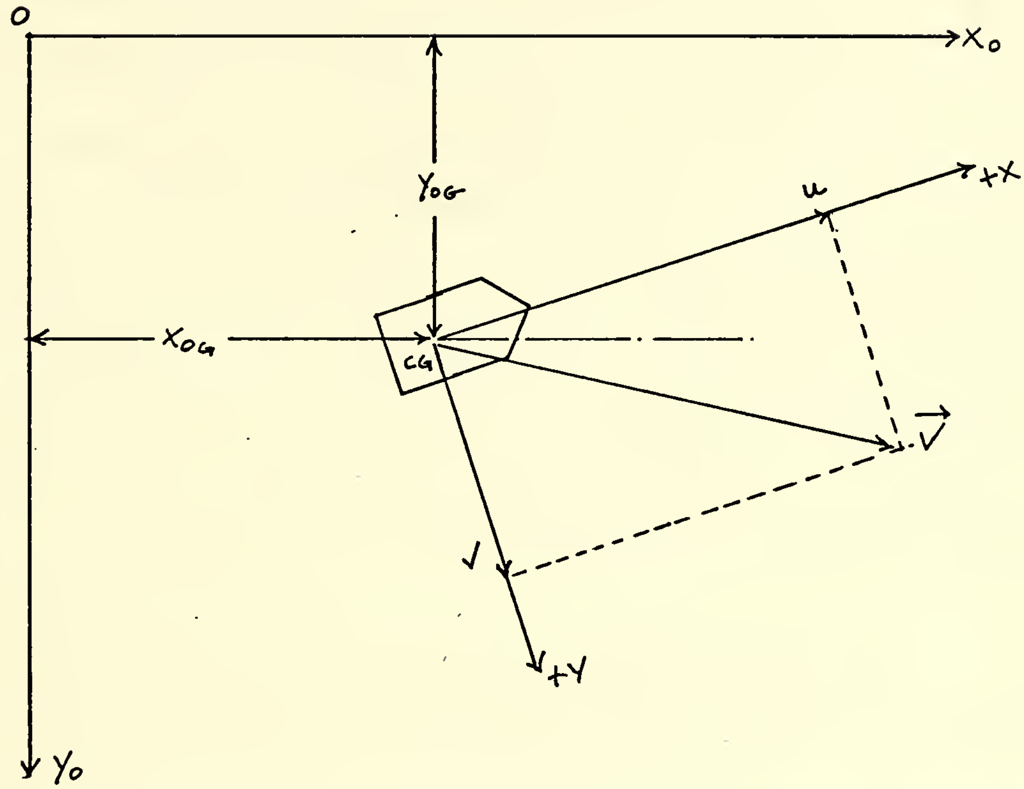


Fig. II-1. Orientation of the Space Axis (X0, Y0) and the Moving Axis (X, Y)

and where the symbols used are respectively

| | |
|---|---|
| m | Mass of the ship |
| X,Y,Z | Components of force in the X,Y,Z directions |
| L,M,N | Components of moment about the X,Y,Z axis |
| U,V,W | Components of velocity in the X,Y,Z directions |
| $x_G, y_G, z_G$ | Distances from the center of gravity to the origin in the X,Y,Z directions |
| P,Q,R | Components of the angular velocity about the X,Y,Z axis |
| $I_x, I_y, I_z$ | Moments of inertia about the X,Y,Z axis |

Equations II-2 describe the reaction of the rigid body to applied forces as a function of the geometric and physical characteristics of the body itself. They do not include any of the applied external forces such as propeller thrust, rudder forces, forces and moments due to the fins (if any), reaction forces of the fluid (hydrodinamic forces), and waves and wind forces.

1. Linearization of the Horizontal Plane Motion Equations

Under the assumption of calm waters, roll, pitch and heave are all negligible, i.e.,

$$P = \dot{P} = Q = \dot{Q} = W = \dot{W} = 0$$

Hence equations (II-2) reduce to

$$\begin{aligned} X &= m\left[\dot{U} - RV - x_G R^2 - y_G \dot{R}\right] \\ Y &= m\left[\dot{V} + UR + x_G \dot{R} - y_G R^2\right] \\ N &= \dot{R} I_z + m\left[x_G(\dot{V} + RU) - y_G(\dot{U} - RV)\right] \end{aligned} \qquad \text{(II-3)}$$

and assuming the coordinate axis origin placed at the center of gravity, $x_G = y_G = 0$, equations II-3 become

$$\begin{aligned} X &= m\left[\dot{U} - RV\right] \\ Y &= m\left[\dot{V} + UR\right] \\ N &= \dot{R} I_z \end{aligned} \qquad \text{(II-4)}$$

The left hand sides of equations (II-4) represent the forces and moments along and about the coordinate axes, and the right hand sides show the corresponding dynamic reaction.

X,Y, and N can be expressed as functions of properties of the body, properties of the fluid and motion, considering for the moment that no controls are applied. On the horizontal plane, no forces or moments are due to orientation changes; the relations are of the form

$$(X,Y,N) \sim f(U,V,R,\dot{U},\dot{V},\dot{R},\ddot{U},\ddot{V},\ddot{R},\ldots)$$

Considering the ship in an equilibrium condition, here defined by a steady forward velocity,

$U_0 =$ Constant

$V_0 = 0$

$\Psi_0 = 0$

Where

$\Psi =$ Yaw angle,

$\dot{\Psi} = R$

From this point and on in this work only small perturbation of the variables, and, eventually in applied controls, will be under consideration. The instantaneous values of U,V,R and $\Psi$ can be expressed by

$$U = U_0 + u$$
$$V = V_0 + v$$
$$R = R_0 + r$$
$$\Psi = \Psi_0 + \psi$$

The right hand sides of equations (II-4) become

$$X = m\dot{U}$$
$$Y = m[\dot{v} + \psi U_0]$$
$$N = I_z \ddot{\psi} = I_z \dot{r} \qquad \text{(II-5)}$$

Since $\dot{V}_o = \dot{\Psi}_o = 0$ and the second order terms are negligible compared with the first order.

The hydrodynamic forces and moments for these particular motions have been found to be [10]

$$X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial \dot{u}} \triangleq X_u \Delta u + X_{\dot{u}}\dot{u}$$

$$Y = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r} \triangleq Y_v v + Y_{\dot{v}}\dot{v} + Y_r r + Y_{\dot{r}}\dot{r}$$

$$N = \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r} \triangleq N_v v + N_{\dot{v}}\dot{v} + N_r r + N_{\dot{r}}\dot{r}$$

(II-6)

Where the symbols used are defined in Table II-1. The derivatives $X_v$, $X_{\dot{v}}$, $X_r$, $X_{\dot{r}}$, $Y_u$, $Y_{\dot{u}}$, $N_u$ and $N_{\dot{u}}$ vanish for any ship with symmetry about the X-Z plane (starboard-port); this has the effect of decoupling u from v and $\psi$ ; and the remaining cross coupled terms $Y_r$, $Y_{\dot{r}}$, $N_v$, $N_{\dot{v}}$, even though they have small non-zero values, have to be included unless the ship is symmetrical about the Y-Z plane, which is not the usual case.

Substitution of equations (II-6) in (II-5), with

$$\Delta u = U_o - u$$

gives

$$(X_{\dot{u}} - m)\dot{u} + X_u(U_o - u) = 0$$

$$(Y_{\dot{v}} - m)\dot{v} + Y_v v + (Y_r - mU_o)r + Y_{\dot{r}}\dot{r} = 0$$

$$(N_{\dot{r}} - I_z)\dot{r} + N_r r + N_{\dot{v}}\dot{v} + N_v v = 0 \qquad \text{(II-7)}$$

which are the linearized equations of motion in the horizontal plane.

2. Nondimensionalization

For computer simulation purposes, equations (II-7) will be used with the nondimensional coefficients of a Mariner ship, which characteristics are those of Table II-2. The nondimensional coefficients and conversion factors are shown in Table II-3 [2].

TABLE II-1

SYMBOLS AND NOMENCLATURE

| Symbol | Definition |
|---|---|
| $X_{\dot{u}}$ | Derivative of longitudinal force component with respect to longitudinal acceleration component $\dot{u}$. |
| $X_u$ | Derivative of longitudinal force component with respect to longitudinal velocity component u. |
| $Y_v$ | Derivative of lateral force component with respect to transverse velocity component v. |
| $Y_{\dot{v}}$ | Derivative of lateral force component with respect to transverse acceleration component $\dot{v}$. |
| $Y_r$ | Derivative of lateral force component with respect to angular velocity component r. |
| $Y_{\dot{r}}$ | Derivative of lateral force component with respect to angular acceleration component $\dot{r}$. |
| $Y_\delta$ | Derivative of lateral force component with respect to rudder angle component $\delta$. |
| $N_v$ | Derivative of yawing moment component with respect to transverse velocity component v. |
| $N_{\dot{v}}$ | Derivative of yawing moment component with respect to transverse velocity acceleration $\dot{v}$. |
| $N_r$ | Derivative of yawing moment component with respect to angular velocity component r. |
| $N_{\dot{r}}$ | Derivative of yawing moment component with respect to angular acceleration component $\dot{r}$. |
| $N_\delta$ | Derivative of yawing moment component with respect to rudder angle component $\delta$. |
| $r$ | Yawing angular velocity component. |
| $\dot{r}$ | Yawing angular acceleration component. |
| $u_1$ | Initial velocity of origin of body axes relative to fluid. |
| $v$ | Transverse velocity component of origin of ship axes relative to fluid. |
| $\dot{v}$ | Transverse acceleration component of ship axes relative to fluid. |

TABLE II-1 cont'd

| Symbol | Definition |
| --- | --- |
| X | Hydrodynamic longitudinal force (positive direction forward). |
| Y | Hydrodynamic lateral force (positive direction to starboard). |

TABLE II-2

CHARACTERISTICS OF MARINER-TYPE STUDY SHIP

| | |
| --- | --- |
| Length, ft | 527.8 |
| Beam, ft | 76.0 |
| Draft, ft | 29.75 |
| Displacement, tons | 16,800 |
| Block Coefficient, $C_b$ | 0.6 |

TABLE II-3

NONDIMENSIONAL HYDRODYNAMIC COEFFICIENTS
NUMERICAL VALUES AND CONVERSION FACTORS

| Nondimensional Coefficient | Nondimensionalizing Factor | Nondimensional Value x $10^5$ |
|---|---|---|
| $(X'_{\dot{u}} - m')$ | $\frac{1}{2}\rho L^3$ | -850 |
| $X'u$ | $\frac{1}{2}\rho L^2 u_1$ | -120 |
| $Y'v$ | $\frac{1}{2}\rho L^2 u_1$ | -1243 |
| $(Y'_{\dot{v}} - m')$ | $\frac{1}{2}\rho L^3$ | -1500 |
| $(Y'r - m')$ | $\frac{1}{2}\rho L^3 u_1$ | -510 |
| $Y'_{\dot{r}} - m'x'_G)$ | $\frac{1}{2}\rho L^4$ | -27 |
| $Y'_\delta$ | $\frac{1}{2}\rho L^2 u_1^2$ | 270 |
| $N'_v$ | $\frac{1}{2}\rho L^3 u_1$ | -351 |
| $N'_{\dot{v}}$ | $\frac{1}{2}\rho L^4$ | -19.7 |
| $(N'_r - m'x'_G)$ | $\frac{1}{2}\rho L^4 u_1$ | -227 |
| $(N'_{\dot{r}} - I'_z)$ | $\frac{1}{2}\rho L^5$ | -68 |
| $N'_\delta$ | $\frac{1}{2}\rho L^3 u_1^2$ | -126 |
| $X'_n$ | $\frac{1}{2}\rho L^3 u_1$ | 4.62 |
| $Y'_n$ | $\frac{1}{2}\rho L^3 u_1$ | -0.52 |
| $N'_n$ | $\frac{1}{2}\rho L^4 u_1$ | 0.26 |
| $X'_\delta$ | $\frac{1}{2}\rho L^3 u_1$ | 0.00 |

Note: $x_G = 0$ $\rho$ = Sea water density (lb/ft)

$L$ = Ship's length (ft)

$u_1$ = Initial velocity of the body axes relative to fluid (ft/sec)

In order to simplify the notation, no special symbols will be used for the nondimensional quantities; it is well understood that only these quantities are being concerned.

Taking the initial velocity of the origin of the body axis relative to the fluid as the nondimensionalizing factor for velocities comes

$$U_0 = 1$$

and equations (II-7) are written in nondimensional form as

$$\begin{aligned}
&(X_{\dot{u}} - m) + X_u (u-1) = 0 \\
&(Y_{\dot{v}} - m) + Y_v v + (Y_r - m) r + Y_{\dot{r}} \dot{r} = 0 \\
&(N_{\dot{r}} - I_z) \dot{r} + N_r r + N_{\dot{v}} \dot{v} + N_v v = 0
\end{aligned} \qquad \text{(II-8)}$$

3. Computer Simulation

If the motion of the ship is to be considered under external perturbations and with acting controls, equations (II-8) must include terms expressing forces and moments due to sea and wind excitations, and forces and moments caused by rudder or movable fins deflections.

The rudder and fins forces and moments are considered control elements; all other forces and moments are not normally controllable inputs, but they must be included in cases where the ship has to be controlled in their presence. To this category belong the iteractive forces and moments generated in the case of ships in close underway replenishment stations, as it will be seen in part B of this section.

Considering the rudder as the only control input, equations (II-8) become

$$\begin{aligned}
&(X_{\dot{u}} - m) \dot{u} + X_u (u-1) + X_\delta \delta = 0 \\
&(Y_{\dot{v}} - m) \dot{v} + Y_v v + (Y_r - m) r + Y_{\dot{r}} \dot{r} + Y_\delta \delta = 0 \\
&(N_{\dot{r}} - I_z) \dot{r} + N_r r + N_{\dot{v}} \dot{v} + N_v v + N_\delta \delta = 0
\end{aligned} \qquad \text{(II-9)}$$

where

$\delta$ = rudder deflection angle, measured from the XZ plane of the ship to the plane of the rudder.

$X_\delta, Y_\delta, N_\delta$ = first derivative of rudder forces and moments, with values given in Table II-4 [2].

TABLE II-4

NONDIMENSIONAL FACTORS AND VALUES

| Parameter | Nondimensionalizing Factor | Nondimensional Value $10^{-5}$ |
|---|---|---|
| $X_\delta$ | $\frac{1}{2}\rho L^2 u_1^2$ | 0.0 |
| $Y_\delta$ | $\frac{1}{2}\rho L^2 u_1^2$ | 270. |
| $N_\delta$ | $\frac{1}{2}\rho L^3 u_1^2$ | -126. |

Note: $\rho$, L, $u_1$, as given in Table II-3.

Taking the Laplace transform of equations (II-9), and considering that $X_\delta = 0$, as given by Table I-A,

$$u(s)\left[s(m - X_{\dot u}) - X_u\right] + \frac{X_u}{s} = 0$$

$$v(s)\left[s(m - Y_{\dot v}) - Y_v\right] + r(s)\left[-sY_{\dot r} + (m - Y_r)\right] = Y_\delta\,\delta(s)$$

$$v(s)\left[-sN_{\dot v} - N_v\right] + r(s)\left[s(I_z - N_{\dot r}) - N_r\right] = N_\delta\,\delta(s)$$

(II-10)

or

$$\frac{v(s)}{s}\left[s^2(m - Y_{\dot v}) - Y_v s\right] + \frac{r(s)}{s}\left[-s^2Y_{\dot r} + s(m - Y_r)\right] = Y_\delta\,\delta(s)$$

$$\frac{v(s)}{s}\left[-s^2N_{\dot v} - sN_v\right] + \frac{r(s)}{s}\left[s^2(I_z - N_{\dot r}) - sN_r\right] = N_\delta\,\delta(s)$$

$$\frac{u(s)}{s}\left[s^2(m - X_{\dot u}) - sX_u\right] = -\frac{X_u}{s}$$

(II-11)

letting

$a_{11} = m - Y_{\dot v}$

$b_{11} = -Y_v$

$c_{11} = 0$

$a_{21} = -Y_{\dot{r}}$

$b_{21} = m - Y_r$

$c_{21} = 0$

$a_{12} = -N_{\dot{v}}$

$b_{12} = -N_v$

$c_{12} = 0$

$a_{22} = I_z - N_{\dot{r}}$

$b_{22} = -N_r$

$c_{22} = 0$

$a_{33} = m - X_{\dot{u}}$

$b_{33} = -X_u$

$c_{33} = 0$

Equations (II-11) can be written as

$$\frac{V(s)}{s}\left[a_{11}s^2 + b_{11}s + c_{11}\right] + \Psi(s)\left[a_{12}s^2 + b_{12}s + c_{12}\right] = Y_\delta\,\delta(s)$$

$$\frac{V(s)}{s}\left[a_{21}s^2 + b_{21}s + c_{21}\right] + \Psi(s)\left[a_{22}s^2 + b_{22}s + c_{22}\right] = N_\delta\,\delta(s)$$

$$\frac{U(s)}{s}\left[a_{33}s^2 + b_{33}s + c_{33}\right] = 0 \qquad \text{(II-12)}$$

Setting

$$\frac{V(s)}{s} = A(s) \qquad v = \dot{A}$$

$$\Psi(s) = B(s) \qquad \psi = B$$

$$\frac{U(s)}{s} = C(s) \qquad u = \dot{C}$$

$$IF1 = Y_\delta\,\delta(s) = KA1 \cdot D1$$

$$IF2 = N_\delta\,\delta(s) = KB1 \cdot D1$$

$$IF3 = -\frac{X_u}{s}$$

Equations (II-12) become

$$a_{11}\ddot{A} + b_{11}\dot{A} + c_{11}A + a_{21}\ddot{B} + b_{21}\dot{B} + c_{21}B = IF1$$
$$a_{12}\ddot{A} + b_{12}\dot{A} + c_{12}A + a_{22}\ddot{B} + b_{22}\dot{B} + c_{22}B = IF2$$
$$a_{33}\ddot{C} + b_{33}\dot{C} + c_{33}C = IF3 \qquad \text{(II-13)}$$

or simply

$$a_{11}\ddot{A} + a_{21}\ddot{B} = I1$$
$$a_{12}\ddot{A} + a_{22}\ddot{B} = I2$$
$$a_{33}\ddot{C} = I3 \qquad \text{(II-14)}$$

where

$$I1 = -b_{11}\dot{A} - c_{11}A - b_{21}\dot{B} - c_{21}B + IF1$$
$$I2 = -b_{12}\dot{A} - c_{12}A - b_{22}\dot{B} - c_{22}B + IF2$$
$$I3 = -b_{33}\dot{C} - c_{33}C + IF3$$

Solution of Equations (II-14) yields

$$\ddot{A} = \frac{\begin{vmatrix} I_1 & a_{21} & 0 \\ I_2 & a_{22} & 0 \\ I_3 & 0 & a_{33} \end{vmatrix}}{\Delta}, \qquad \ddot{B} = \frac{\begin{vmatrix} a_{11} & I_1 & 0 \\ a_{12} & I_2 & 0 \\ 0 & I_3 & a_{33} \end{vmatrix}}{\Delta}, \qquad \ddot{C} = \frac{\begin{vmatrix} a_{11} & a_{21} & I_1 \\ a_{12} & a_{22} & I_2 \\ 0 & 0 & I_3 \end{vmatrix}}{\Delta} \qquad \text{(II-15)}$$

with

$$\Delta = \begin{vmatrix} a_{11} & a_{21} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} = a_{33}(a_{11}a_{22} - a_{12}a_{21})$$

and replacing the relations between A, B, C and the original variables

$v$, $\psi$, u,

$$v = \dot{A} = v_0 + \int \ddot{A}\,dt$$
$$\psi = B = \psi_0 + \int \dot{B}\,dt = \psi_0 + \int \left[\dot{B}(0) + \int \ddot{B}\,dt\right] d\tau$$
$$u = \dot{C} = u_0 + \int \ddot{C}\,dt \qquad \text{(II-16)}$$

The transformation from ship to space coordinate system is defined by the following relations, obtained from Figure II-1:

$$\dot{y} = u \sin \psi + v \cos \psi$$
$$\dot{x} = u \cos \psi - v \sin \psi \tag{II-17}$$

and give

$$y = y_0 + \int \dot{y}\, dt$$
$$x = x_0 + \int \dot{x}\, dt \tag{II-18}$$

Equations (II-14) through (II-18) were translated into DSL/360 Computer Program I. With a constant rudder deflection $\delta$ = D1 = 0.1, the results are shown in Figures II-2 (yaw angle versus time) and Figure II-3 (sway versus surge), the characteristic turning radius of the ship.

4. Stability Investigation

The stability test determines whether or not the ship returns to an established equilibrium condition (straight ahead motion at constant speed), after removing the small disturbance which caused its departure from that equilibrium. A dynamically unstable ship cannot maintain straight line motion when the rudder is amidships. The behavior of the ship can be analyzed by considering either some introduced disturbance and zero control input ($\delta$ = 0) or the control acting as disturbance. For the first case, and neglecting the surge equation because steady forward motion is assumed, equations (II-12) reduce to

$$v(s)(a_{11}s + b_{11}) + r(s)(a_{21}s + b_{21}) = 0$$
$$v(s)(a_{12}s + b_{12}) + r(s)(a_{22}s + b_{22}) = 0 \tag{II-13}$$

yielding the characteristic equation

$$\begin{vmatrix} a_{11}s + b_{11} & a_{21}s + b_{21} \\ a_{12}s + b_{12} & a_{22}s + b_{22} \end{vmatrix} = 0$$

or

$$(a_{11}a_{22} - a_{12}a_{21})s^2 + (a_{11}b_{22} + a_{22}b_{11} - a_{12}b_{21} - a_{21}b_{12})s + (b_{11}b_{22} - b_{12}b_{21}) = 0$$

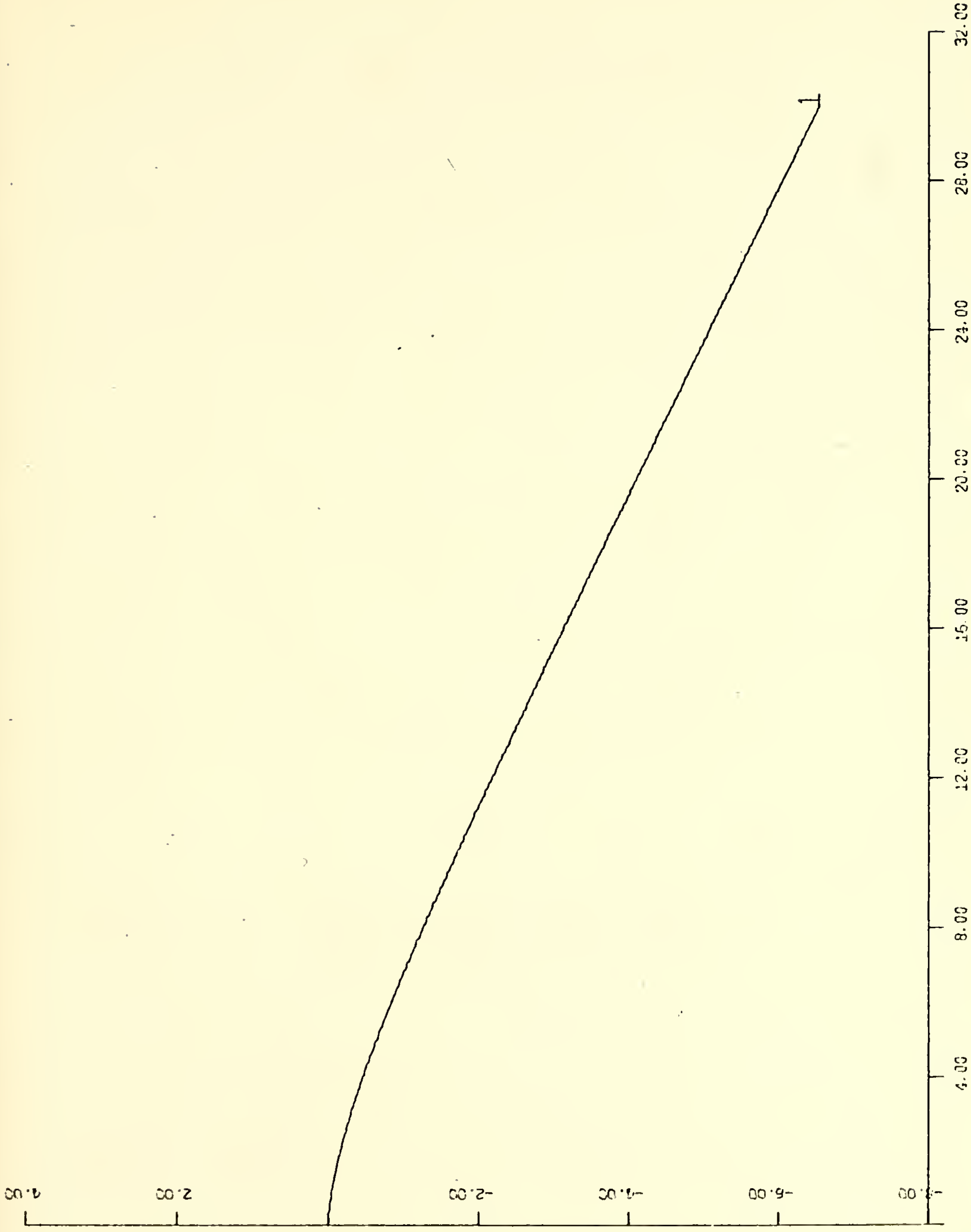


Fig. II-2. Linear Response - Yaw vs. Time D = 0.1

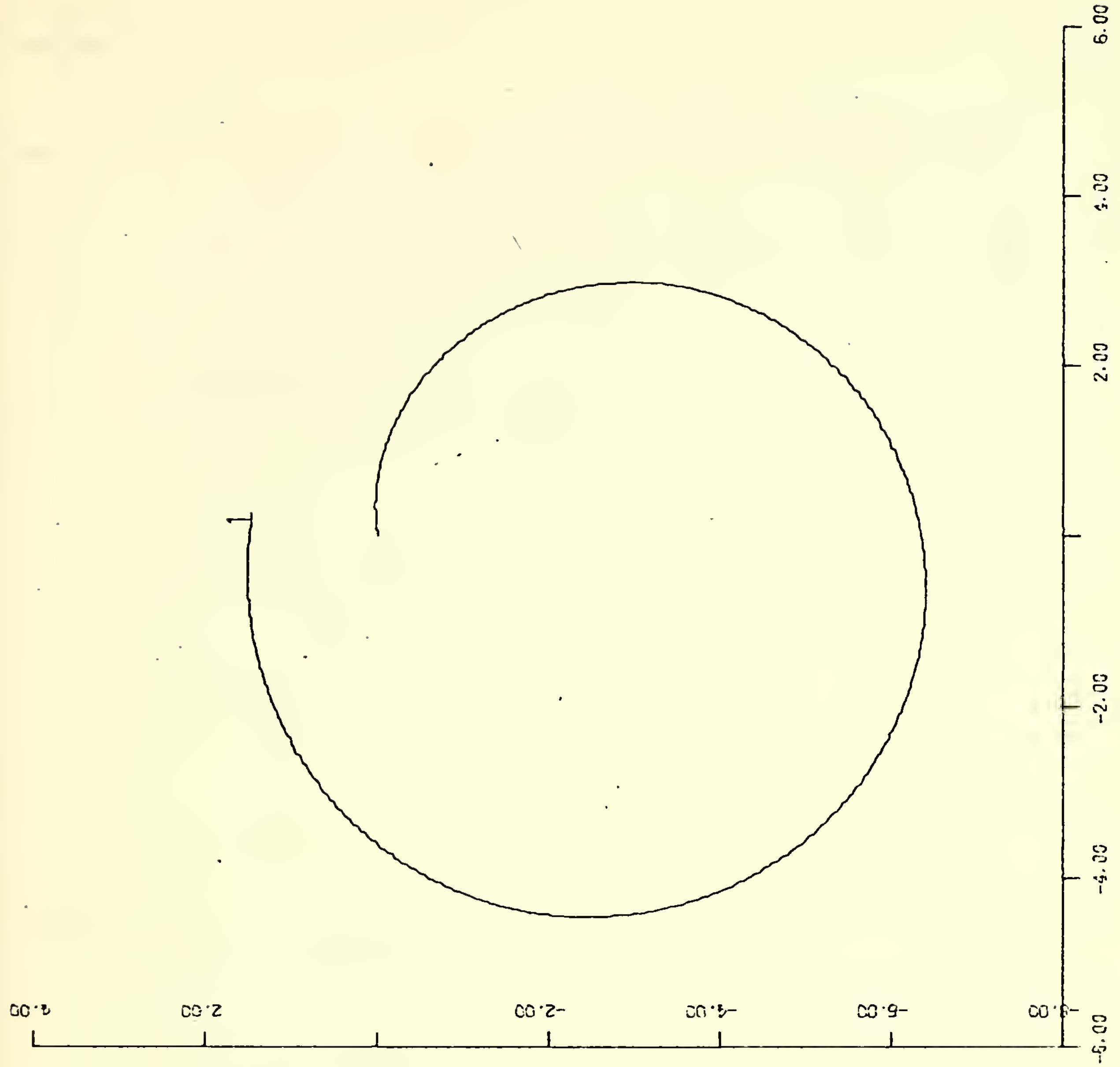

Fig. II-3. Linear Response - Surge vs. Sway
The Turning Radius D = 0.1

replacing values and rearranging,

$$s^2 + 0.685\,s + 1.016 = 0$$

Both roots belong to the left half s - plane; the ship possesses fixed control stability with characteristics

$$\omega_n = 1.008$$

$$\zeta = 0.34$$

5. <u>The Transfer Functions</u>

Defining

$$K_{11} = a_{11}s^2 + b_{11}s$$

$$K_{21} = a_{21}s^2 + b_{21}s$$

$$K_{12} = a_{12}s^2 + b_{12}s$$

$$K_{22} = a_{22}s^2 + b_{22}s$$

$$K_{33} = a_{33}s^2 + b_{33}s$$

equations (II-12) can be written as

$$\frac{V(s)}{s} K_{11} + \Psi(s) K_{21} = Y_\delta\, \delta(s)$$

$$\frac{V(s)}{s} K_{12} + \Psi(s) K_{22} = N_\delta\, \delta(s)$$

$$\frac{u(s)}{s} K_{33} = -\frac{X_u}{s}$$

(II-20)

solving for $V(s)$, $\Psi$ (s), and u(s),

$$\frac{\frac{V(s)}{s}}{\delta(s)} = \frac{\begin{vmatrix} Y_\delta & K_{21} & 0 \\ N_\delta & K_{22} & 0 \\ 0 & 0 & K_{33} \end{vmatrix}}{\Delta}, \quad \frac{\Psi(s)}{\delta(s)} = \frac{\begin{vmatrix} K_{11} & Y_\delta & 0 \\ K_{12} & N_\delta & 0 \\ 0 & 0 & K_{33} \end{vmatrix}}{\Delta}$$

and

$$\frac{u(s)}{\delta(s)} = 0 \tag{II-21}$$

where

$$\Delta = \begin{vmatrix} K_{11} & K_{21} & 0 \\ K_{12} & K_{22} & 0 \\ 0 & 0 & K_{33} \end{vmatrix} = K_{33}\left(K_{11}K_{22} - K_{12}K_{21}\right)$$

or replacing the K's

$$\Delta = s(a_{33}+b_{33})\left[s^2(a_{11}a_{22}-a_{12}a_{21}) + s(a_{11}b_{22}+a_{22}b_{11} - a_{12}b_{12}-a_{21}b_{21}) + (b_{11}b_{22}-b_{12}b_{21})\right]$$

Evaluating the solutions defined by equations (II-21),

$$\frac{v(s)}{\delta(s)} = \frac{K_v\ (s+z_v)}{s^2+ps+q}$$

$$\frac{\psi(s)}{\delta(s)} = \frac{K_r\ (s+z_r)}{s(s^2+ps+q)} \tag{II-22}$$

where

$$K_v = \frac{Y_\delta a_{22} - N_\delta a_{21}}{a_{11}a_{22}-a_{12}a_{21}}, \qquad z_v = \frac{Y_\delta b_{22} - N_\delta b_{21}}{Y_\delta a_{22} - N_\delta a_{21}}$$

$$K_r = \frac{N_\delta a_{11} - Y_\delta a_{12}}{a_{11}a_{22}-a_{12}a_{21}}, \qquad z_r = \frac{N_\delta b_{11} - Y_\delta b_{12}}{N_\delta a_{11} - Y_\delta a_{12}}$$

$$p = \frac{a_{12}b_{12}+a_{22}b_{11}-a_{12}b_{21}-b_{12}a_{21}}{a_{11}a_{22}-a_{12}a_{21}}, \qquad q = \frac{b_{11}b_{22}-b_{12}b_{21}}{a_{11}a_{22}-a_{12}a_{21}} \tag{II-23}$$

The transfer functions defined by equations (II-22) and (II-23), together with the coordinate transformation given by equations (II-17) and (II-18) lead to the block diagram representation of the ship, Figure II-4.

The numerical values are those of Table II-5.

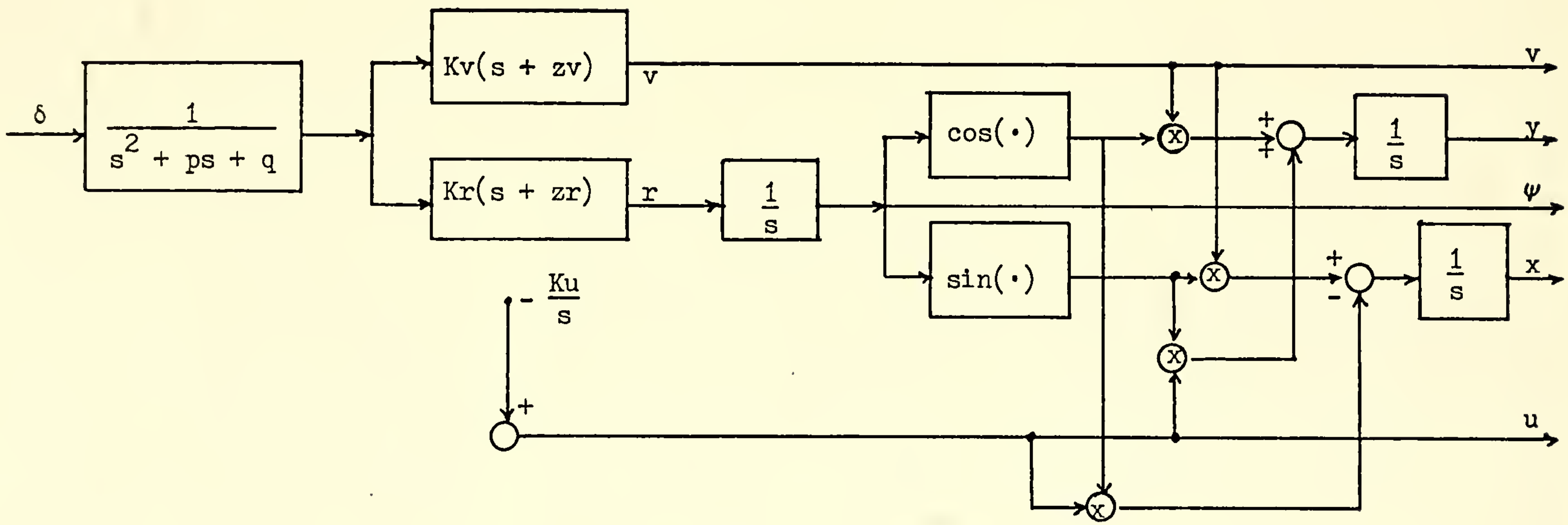


Fig. II-4. Open Loop System Block Diagram For One Ship

[illegible]

TABLE II-5

NUMERICAL VALUES FOR THE TRANSFER FUNCTIONS

$$K_v = 0.21447$$
$$K_r = -1.91507$$

$$Z_v = 5.76923$$
$$Z_r = 1.29369$$

$$p = 0.68467$$
$$q = 1.01659$$

## B. THE MIMO CASE

As stated in part A-3 of this section, equations (II-9) do not consider iterative forces and moments acting on a ship, by effect of other ship maneuvering at short relative distances.

During Newton's experiment [11], for each position of one ship relative to the other, two forces $F_1$ and $F_2$, and moment M were measured. To be coherent with the equations of motion, the resultant of $F_1$ and $F_2$, and M, must be applied on and about the origin of the ship's coordinate axes, i.e., the center of gravity.

Figures II-5 and II-6 [2] show the steady state interaction curves for two similar ships travelling at 15 knots at different parallel positions. The curves for $\Delta y = 50$ and $\Delta x = 100$ ft were determined from experimental data and other curves by interpolation [3].

To include such force and moment in the linear model, the equilibrium condition is redefined as

$\mathcal{U}o$, $F(\Delta xo, \Delta yo)$ and $M(\Delta xo, \Delta yo)$, constants;

$\delta o$ such that $Vo = 0$ and $\Psi o = 0$

[illegible]

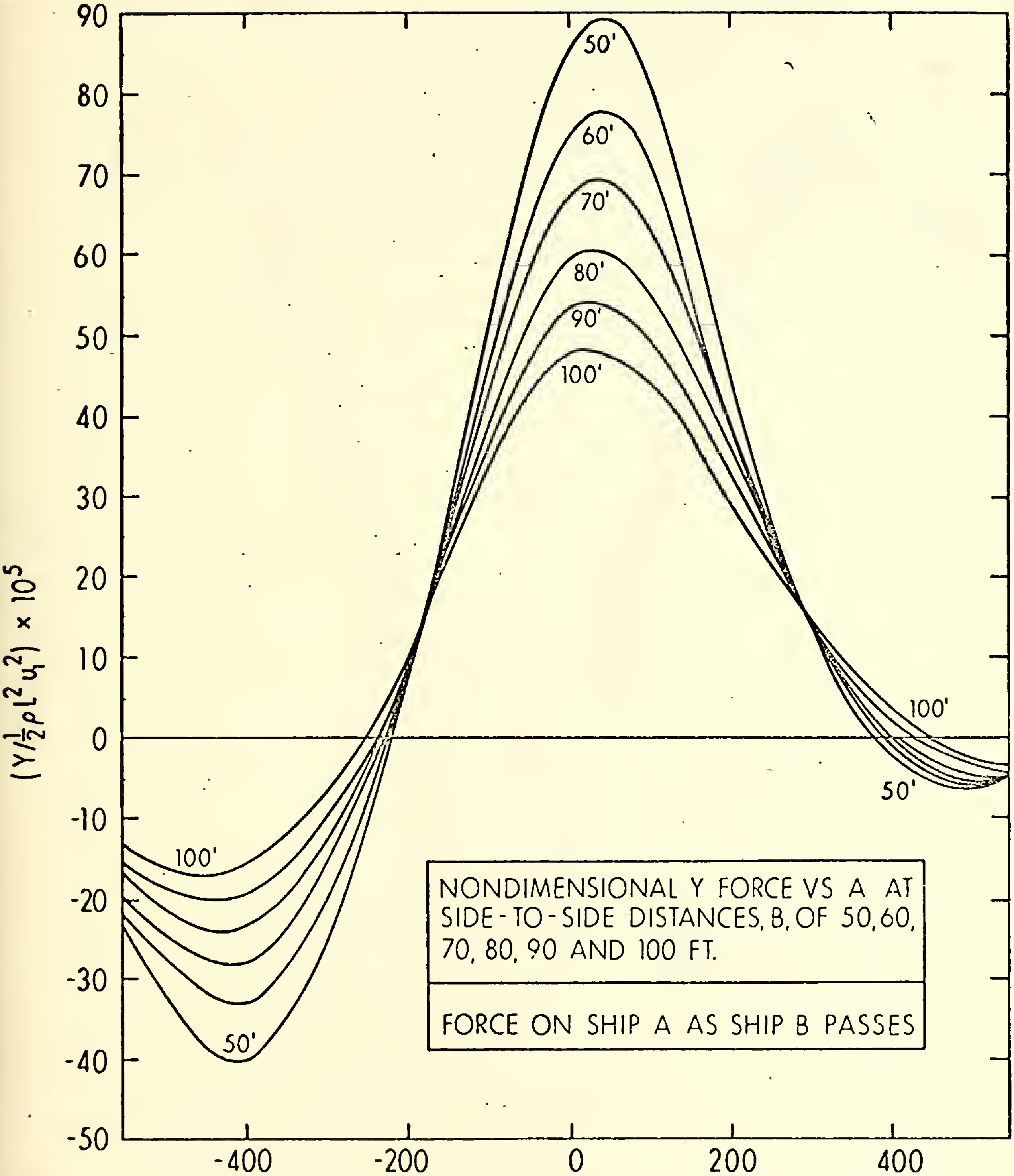


Fig. II-5. Dimensionless Force vs. Longitudinal Separation (ft)

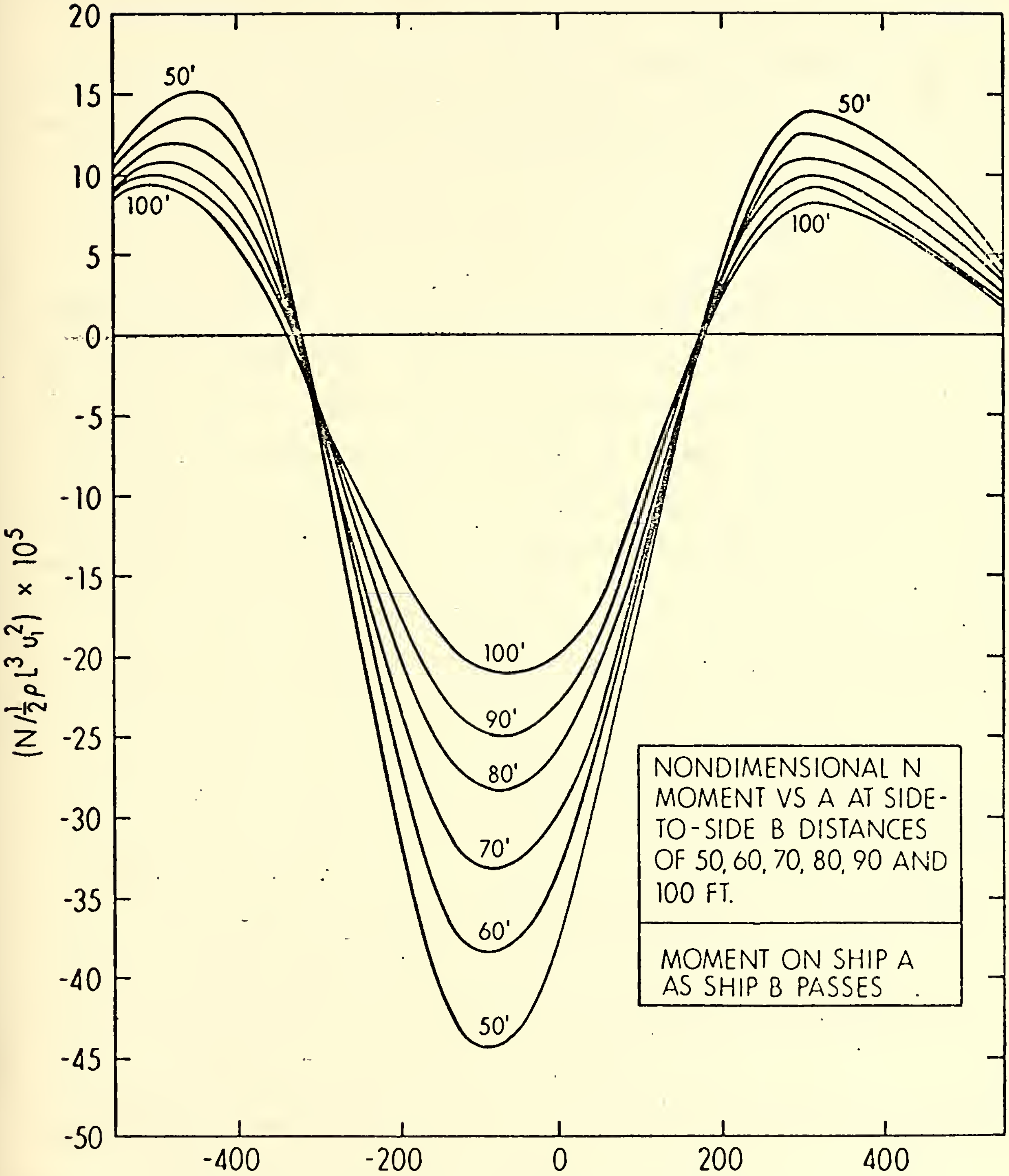


Fig. II-6. Dimensionless Moment vs. Longitudinal Separation (ft)

i.e., it is assumed that a certain amount of rudder angle is introduced to exactly compensate the effects of $\partial F(\Delta x_o, \Delta y_o), \partial M(\Delta x_o, \Delta y_o)$.

Neither $\partial F(\Delta x, \Delta y)$ nor $\partial M(\Delta x, \Delta y)$ can be expressed analytically in a simple form. Then it will be assumed that the linearization of both about the equilibrium condition give expressions of type

$$\partial F(\Delta x, \Delta y) = k_1 \Delta y + k_2 \Delta x$$

$$\partial M(\Delta x, \Delta y) = q_1 \Delta y + q_2 \Delta x \qquad \text{(II-24)}$$

where $k_1$, $k_2$, $q_1$, $q_2$ represent the rate of change of F and M with respect to $\Delta x$ and $\Delta y$ measured at $\Delta x_o$ and $\Delta y_o$, the longitudinal and lateral separations for the established equilibrium condition.

The linear expressions of $\partial F(\Delta x, \Delta y), \partial M(\Delta x, \Delta y)$ must be defined in terms of the variables u, v, $\psi$ and then added to the left hand sides of equations (II-11) under the following assumptions:

a. The two ships are identical;

b. All hydrodynamic coefficients are not affected by the intermingling of the water pressure between the ships, and the motions of the ships, therefore remaining constant;

c. The two ships are considered already as being alongside each other ($\Delta x_o = 0$) ;

d. The forces and moments acting on the ships are equal in magnitude and of opposite sign;

e. The change in the forward velocity will be considered negligible for all practical purposes.

The increments $\Delta x$ and $\Delta y$ can be expressed as given by equations (II-18)

$$\Delta x = x_1 - x_2 = \int (\dot{x}_1 - \dot{x}_2)\, dt$$

$$\Delta y = y_1 - y_2 = \int (\dot{y}_1 - \dot{y}_2)\, dt$$

where the subscript 1 stands for the leading ship and 2 for the tracking ship.

Using equations (II-17),

$$\dot{X}_1-\dot{X}_2=(u_1\cos\Psi_1-v_1\sin\Psi_1)-(u_2\cos\Psi_2-v_2\sin\Psi_2)$$

$$\dot{Y}_1-\dot{Y}_2=(u_1\sin\Psi_1+v_1\cos\Psi_1)-(u_2\sin\Psi_2+v_2\cos\Psi_2) \tag{II-25}$$

For the small perturbations being considered,

$$\cos\Psi_1\approx 1 \qquad \sin\Psi_1\approx 0$$

$$\cos\Psi_2\approx 1 \qquad \sin\Psi_2\approx 0$$

and equations (II-25) reduce to

$$\dot{X}_1-\dot{X}_2\approx 0$$

$$\dot{Y}_1-\dot{Y}_2\approx v_1-v_2$$

and finally

$$\Delta x\simeq 0 \qquad \Delta x(s)=0$$

$$\Delta y\simeq\int(v_1-v_2)dt \qquad \Delta y(s)=\frac{1}{s}(v_1-v_2)$$

Then equations (II-12) are modified to include the interaction effects and become

$$\frac{v_1(s)}{s}\left[a_{11}s^2+b_{11}s+c_{11}\right]+\Psi_1(s)\left[a_{12}s^2+b_{12}s+c_{12}\right]-\frac{v_2(s)}{s}k=Y_\delta\delta_1(s)$$

$$\frac{v_1(s)}{s}\left[a_{21}s^2+b_{21}s+c_{21}\right]+\Psi_1(s)\left[a_{22}s^2+b_{22}s+c_{22}\right]-\frac{v_2(s)}{s}q=N_\delta\delta_1(s)$$

$$\frac{v_2(s)}{s}\left[a_{11}s^2+b_{11}s+c_{11}\right]+\Psi_2(s)\left[a_{12}s^2+b_{12}s+c_{12}\right]-\frac{v_1(s)}{s}k=Y_\delta\delta_2(s)$$

$$\frac{v_2(s)}{s}\left[a_{21}s^2+b_{21}s+c_{21}\right]+\Psi_2(s)\left[a_{22}s^2+b_{22}+c_{22}\right]-\frac{v_1(s)}{s}q=N_\delta\delta_2(s)$$

(II-26)

where now

$c_{11}=k$ and $c_{21}=q$

letting

$$
\begin{aligned}
p_1 &= a_{11} s^2 + b_{11} s + c_{11} \\
p_2 &= a_{12} s^2 + b_{12} s \\
p_3 &= a_{21} s^2 + b_{21} s + c_{21} \\
p_4 &= a_{22} s^2 + b_{22} s
\end{aligned}
\tag{II-27}
$$

equations (II-26) become

$$
\begin{aligned}
\frac{V_1(s)}{s} p_1 + \psi_1(s) p_2 - \frac{V_2(s)}{s} k &= Y_\delta \, \delta_1(s) \\
\frac{V_1(s)}{s} p_3 + \psi_1(s) p_4 - \frac{V_2(s)}{s} q &= N_\delta \, \delta_1(s) \\
-\frac{V_1(s)}{s} k + \frac{V_2(s)}{s} p_1 + \psi_2(s) p_2 &= Y_\delta \, \delta_2(s) \\
-\frac{V_1(s)}{s} q + \frac{V_2(s)}{s} p_3 + \psi_2(s) p_4 &= N_\delta \, \delta_2(s)
\end{aligned}
\tag{II-28}
$$

Equations (II-28) show that two ships affected by interaction forces and moments can be described as a multivariable system where the deflection of the rudders $\delta_1$ and $\delta_2$ are the control inputs and the yaw angles $\psi_1$ and $\psi_2$ the outputs of interest.

The remaining part of this section has as objective to determine the form of the entries of the open loop transfer function matrix G, namely

$$
G(s) = \begin{bmatrix} \dfrac{\psi_1}{\delta_1} & \dfrac{\psi_1}{\delta_2} \\[2ex] \dfrac{\psi_2}{\delta_1} & \dfrac{\psi_2}{\delta_2} \end{bmatrix}
$$

so that the system can be analyzed and modified, if necessary, to become steady state decoupled: after a transient period of time, a variation introduced in the rudder angle of one ship will not alter the yaw angle of the other ship.

As an extension of the analysis, compensation will be introduced in order to make the system to achieve some specified performance factors.

1. Determination of the Transfer Function Matrix

Analysis of the transfer function matrix describing the M.I.M.O. system for the purposes of this study does not require more than the determination of the plant type number matrix[1], which provides the required information for designing the compensation that, cascaded with the plant, will decouple its steady states. Cascade compensation is the elected method, vice diagonalization of the matrix transfer function, for being physically realizable, flexible and effective.

The assumptions made for determining the coupled equations (II-28) do not go against the present necessity. In particular, the results obtained by handling those equations indicate, as will be shown next, a reasonable margin of safety that will permit us to relax some of the constraints introduced. Concerning computer simulation, the system can be described by its states and the desired outputs as functions of these states. In essence this problem will be solved by modifying the aforementioned computer program I and using a two-entries table look-up and interpolation subprogram to provide the values of forces and moments for each pair $\Delta x$, $\Delta y$.

---

[1] A type number matrix is, by definition [7] the matrix which entries $t(i,j)$ are given by

where

$t(i,j)$ can be any integer and

$G'(i,j)$ are such that their limits, as $s \to 0$ are non-zero finite constants.

Simply stated, $t(i,j)$ are the powers of $s$ that can be factored without cancellation, in the denominators of each entry of a matrix.

[illegible]

[illegible]

[illegible]

[illegible]

The plant type number matrix is obtained as follows:

Solving (II-28) for $\Psi_1(s)$ and $\Psi_2(s)$ ,

$$\Psi_1(s) = \frac{1}{\Delta}\left[G_{11}\,\delta_1(s) + G_{12}\,\delta_2(s)\right] \tag{II-29}$$

$$\Psi_2(s) = \frac{1}{\Delta}\left[G_{21}\,\delta_1(s) + G_{22}\,\delta_2(s)\right] \tag{II-30}$$

where

$$\Delta = \begin{bmatrix} p_1 & p_2 & -k & 0 \\ p_3 & p_4 & -q & 0 \\ -k & 0 & p_1 & p_2 \\ -q & 0 & p_3 & p_4 \end{bmatrix}, \tag{II-31}$$

$$\Delta = (p_1 p_4 - p_2 p_3)^2 - (k p_4 - q p_2)^2 \tag{II-32}$$

Recalling the definitions of $p_1, p_2, p_3, p_4$ given by equations (II-27), it can be seen that

$p_1$ and $p_3$ are second order polynomals in s satisfying

$$\lim_{s\to 0} p \neq 0$$

Thus the special case of having either k or q identically zero is avoided. Inspection of the curves shown in Figures II-5 and II-6 indicates that:

i) within the range of interest of $\Delta x$, the lateral forces do vary with $\Delta y$ (have non-zero slope k) and so do the lateral moments.

ii) The shape of the curves for the latter allow points where the slope q becomes zero; such points will be considered as singular points and not included on the present appreciation, which agrees with the planned simulation and the real case,

where even passing through a zero value, the rate of change of the lateral moments do not remain constant at zero.

iii) Both the forces and lateral moments approach zero as the lateral separation $\Delta y$ increases; this is the case of the s.i.s.o. system analyzed in part II-A.

In equation (II-29),

$$\psi_1(s) = \frac{(N_\delta p_1 - Y_\delta p_3)(p_1 p_4 - p_2 p_3) + (N_\delta k - Y_\delta q)(q p_2 - k p_4)}{\Delta}\delta_1(s) + $$
$$+ \frac{(Y_\delta p_4 - N_\delta p_2)(q p_1 - k p_3)}{\Delta}\delta_2(s) \tag{II-33}$$

where $\Delta$ is given by equation (II-32).

Replacing $p_1, p_2, p_3, p_4$ as indicated by (II-27), and taking separately $G_{11}(s)$ and $G_{12}(s)$,

$$G_{11}(s) = \frac{\psi_1(s)}{\delta_1(s)} = -\frac{s}{\Delta}\Big\{ s^5 [c_3 (N_\delta a_{11} - Y_\delta a_{21}) + s^4 [c_3 (N_\delta b_{11} - Y_\delta b_{21}) +$$
$$+ c_2 (N_\delta a_{11} - Y_\delta a_{12})] + s^3 [c_3 (k - q) + c_2 (N_\delta b_{11} - Y_\delta b_{21}) + c_1 (N_\delta a_{11} - Y_\delta a_{21})] +$$
$$+ s^2 [c_2 (k - q) + c_1 (N_\delta b_{11} - Y_\delta b_{21}) + c_0 (N_\delta a_{11} - Y_\delta a_{21})] +$$
$$+ s\, [c_0 (N_\delta b_{11} - Y_\delta b_{21}) + c_1 (k - q) + m (q a_{12} - k a_{22})] +$$
$$+ c_0 (k - q - m)\Big\}$$

where

$$c_3 = a_{11} a_{22} - a_{12} a_{21}, \qquad c_2 = a_{11} b_{22} + a_{22} b_{11} - a_{12} b_{21} - a_{21} b_{12}$$
$$c_1 = b_{11} b_{22} - b_{12} b_{21} + k a_{22} - q a_{12}, \qquad c_0 = k b_{22} - q b_{21}$$

and

$$m = N_\delta k - Y_\delta q$$

At least the first power of s is factorable in the numerator -- the independent term is zero only if:

a) $C_0 = 0$ or $k = \frac{b_{12}}{b_{21}} q = 1.3 q$

b) $k - q = m$ or $k = \frac{1 - Y_\delta}{1 - N_\delta} q = -2.15 q$

and for the values above the term in s is non-zero.

Only for the set of pairs ( $\Delta$x, $\Delta$ y) where k,q satisfy the relations above, the numerator of $G_{11}(s)$ has the second power of s factorable; in general, only the first power can be separated. The special cases will be put aside for the moment but being aware of their existence one must take care of them in future analysis.

Expanding the denominator,

$$\Delta = s^2\{(c_3 s^3 + c_2 s^2 + c_1 s + c_0)^2 - (d_1 s + c_0)^2\} =$$

$$= s^3\left[c_3^2 s^5 + 2c_2c_3 s^4 + (c_2^2 + 2c_1c_3)s^3 + 2(c_0c_3 + c_1c_2)s^2 + \right.$$
$$\left. + (c_1^2 + 2c_0c_2 - d_1^2)s + 2c_0(c_1 - d_1)\right]$$

where

$$d_1 = ka_{22} - qa_{12}$$

The independent term is zero only for

$$a)\ \frac{k}{q} = \frac{b_{12}}{b_{22}} = 1.3$$

and this value makes the term in s to be non-zero. Except for this special case the highest factorable power of s is $s^3$.

Hence

$$G_{11}(s) = \frac{\Psi_1(s)}{\delta_1(s)}$$

is a type 2 transfer function.

Expanding the numerator of $G_{12}(s)$,

$$N_{G12} = s^2[(Y_\delta a_{22} - N_\delta a_{12})s + (Y_\delta b_{22} - N_\delta b_{12})] \times$$
$$\times [(qa_{11} - ka_{21})s + (qb_{11} - kb_{21})]$$

since

$$Y_\delta b_{22} - N_\delta b_{12} \neq 0$$

it suffices to investigate $qb_{11} - kb_{21}$ to see if a power higher than $s^2$ can be factored.

The independent term is zero only if

$$b)\ \frac{k}{q} = \frac{b_{11}}{b_{21}} = 2.44.$$

Hence, in general,

$$G_{12}(s) = \frac{\psi_1(s)}{\delta_2(s)}$$

is a type 1 transfer function.

The solution of (II-28) for $\psi_2(s)$ is

$$\psi_2(s) = \frac{(N_\delta p_1 - Y_\delta p_3)(p_1 p_4 - p_2 p_3) + (N_\delta k - Y_\delta q)(q p_2 - k p_4)}{\Delta}\,\delta_2(s) +$$
$$+ \frac{(Y_\delta p_4 - N_\delta p_2)(q p_1 - k p_3)}{\Delta}\,\delta_1(s)$$

(II-34)

as could be expected by symmetry, $\psi_2(s)$ has the same form as $\psi_1(s)$.

If in either equation (II-33) or (II-34) if

$k = q = \text{constant} = 0$

the resultant expression is

$$\frac{\psi(s)}{\delta(s)} = \frac{Y_\delta p_3 - N_\delta p_1}{p_1 p_4 - p_2 p_3}$$

and the expansion of $p_1, p_2, p_3, p_4$ yields the very same transfer function (II-22-b), obtained for the S.I.S.O. system. This result could be expected since it was stated that k = q = 0 (except for the aforesaid singularities) would happen only for large relative distances between the ships.

As a conclusion of this section, it was found that the plant type number matrix is in general

$$\underset{\sim}{T_p} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{II-35}$$

## C. STEADY STATE DECOUPLING

The cascade compensator is the most suitable way of decoupling the steady states of a M.I.M.O. system [15]. The criterion used to determine the number of integrators in each entry of the compensator matrix is

summarized in Appendix A so that in this section only its application to the system being studied will be considered.

The closed loop block diagram, including the compensator is shown in Figure II-10.

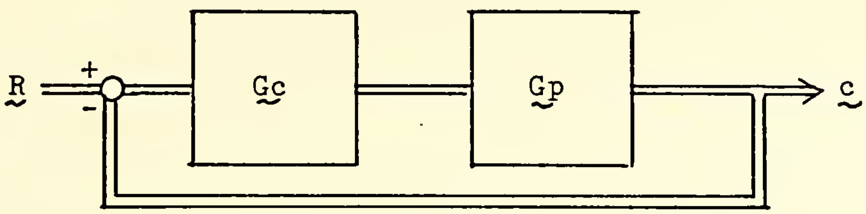


Fig. II-10. Closed Loop Block Diagram

where

$\underset{\sim}{G}p(s)$ is the plant transfer function matrix (2 x 2)

$\underset{\sim}{G}c(s)$ is the compensator transfer function matrix (2 x 2)

$\underset{\sim}{R}(s) = \begin{bmatrix} \delta_1(s) \\ \delta_2(s) \end{bmatrix}$ is the reference input vector (2 x 1)

$\underset{\sim}{C}(s) = \begin{bmatrix} \psi_1(s) \\ \psi_2(s) \end{bmatrix}$ is the output vector (2 x 1)

and, by definition, $\underset{\sim}{G}(s) \triangleq \underset{\sim}{G}p(s) \cdot \underset{\sim}{G}c(s)$ is the open loop transfer function matrix and

$$\underset{\sim}{F}(s) = [\underset{\sim}{I} + \underset{\sim}{G}(s)]^{-1} \underset{\sim}{G}(s) \tag{II-36}$$

$$\underset{\sim}{F}(s) = \underset{\sim}{I} - [\underset{\sim}{I} + \underset{\sim}{G}]^{-1} \tag{II-37}$$

is the closed loop transfer function matrix, and $\underset{\sim}{I}$ is the identity matrix.

By definition [7], a system as shown in Figure II-10 is steady state decoupled if and only if

$$\lim_{s \to 0} \sum_{\substack{q=1 \\ q \neq r}}^{n} \frac{r_{q,kq}}{s^{kq-1}} \cdot \frac{\underset{\sim}{\Delta}(\underset{\sim}{I}+\underset{\sim}{G})_{q,r}}{\underset{\sim}{\Delta}(\underset{\sim}{I}+\underset{\sim}{G})} = 0 \tag{II-38}$$

for all $n \geq r \geq 1$ where

$\Delta(\underset{\sim}{I}+\underset{\sim}{G})$ is the determinant of $(\underset{\sim}{I} + \underset{\sim}{G})$

$\Delta(\underset{\sim}{I}+\underset{\sim}{G})_{qr}$ is the qrth cofactor of $(\underset{\sim}{I} + \underset{\sim}{G})$

$k_q$ is the type number of the qth input, $1 \leq q \leq n$

If $\underset{\sim}{G}_c$ is a diagonal matrix and all inputs are steps ($k_q = 1$), the compensator type number matrix $\underset{\sim}{T}_c$, which gives the number of integrators required for steady state decoupling of a 2 x 2 system is obtained by satisfying the conditions:

$$M \geq N_{12}, N_{21},$$
$$M \geq 0 \tag{II-39}$$

where

$$M \triangleq \text{Max}\left[(t_{c11}+t_{p11}), (t_{c22}+t_{p22}), (t_{c11}+t_{c22}+\text{Det}(\underset{\sim}{T}_p))\right]$$
$$N_{12} \triangleq t_{c11}+t_{p21}$$
$$N_{21} \triangleq t_{c22}+t_{p12} \tag{II-40}$$

from equation (II-35)

$$t_{p11} = t_{p22} = 2$$
$$t_{p12} = t_{p21} = 1$$
$$\text{Det}\ \underset{\sim}{T}_p = 3$$

Then

$$N_{12} = t_{c11} + 1$$
$$N_{21} = t_{c22} + 1$$
$$M = \text{Max}\left\{(t_{c11}+2), (t_{c22}+2), (t_{c11}+t_{c22}+3)\right\}$$

The solution is not unique; however, minimum integers $t_{c11}$ and $t_{c22}$ must be chosen so that the order of the system will not become higher than strictly necessary. In the case above, clearly

$$t_{c11} = t_{c22} = 0$$

are the required values, since

$$N_{12} = N_{21} = 1$$
$$M = 3$$

Therefore no integrators are required in the compensator matrix:

$$\underset{\sim}{T}_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \underset{\sim}{G}_c = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \tag{II-41}$$

are respectively the type number and transfer function matrices of the compensator, where $g_{11}$ and $g_{22}$ are the values of the gains to be introduced in each channel.

By (II-36) and with (II-41), $\underset{\sim}{G}$ will have the same type number matrix as $\underset{\sim}{G}_p$ namely

$$\underset{\sim}{T} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Then $\underset{\sim}{G}$ can be written as

$$\underset{\sim}{G} = \begin{bmatrix} \dfrac{P_{11}(s)}{s^2 P_\Delta(s)} & \dfrac{P_{12}(s)}{s P_\Delta(s)} \\ \\ \dfrac{P_{21}(s)}{s P_\Delta(s)} & \dfrac{P_{22}(s)}{s^2 P_\Delta(s)} \end{bmatrix}$$

Where $P_\Delta(s)$ and $P_{i,j}(s)$, $i,j = 1,2$ are polynominals in s such that

$$\lim_{s \to 0} P(s) \neq 0$$

The results obtained can be readily checked using the definition of steady state decoupling (II-38).

$$\underset{\sim}{I} + \underset{\sim}{G} = \begin{bmatrix} \dfrac{s^2 P_\Delta + P_{11}}{s^2 P_\Delta} & \dfrac{P_{12}}{s P_\Delta} \\ \\ \dfrac{P_{21}}{s P_\Delta} & \dfrac{s^2 P_\Delta + P_{22}}{s^2 P_\Delta} \end{bmatrix}$$

$$\Delta_2 \left[ \underset{\sim}{I} + \underset{\sim}{G} \right] = \frac{(s^2 P_\Delta + P_{11})(s^2 P_\Delta + P_{22}) - s^2 P_{12} P_{21}}{s^4 P_\Delta}$$

$$\Delta_2 \left[ \underset{\sim}{I} + \underset{\sim}{G} \right]_{1,2} = \frac{P_{21}}{s P_\Delta}, \qquad \Delta_2 \left[ \underset{\sim}{I} + \underset{\sim}{G} \right]_{2,1} = \frac{P_{12}}{s P_\Delta}$$

$$\lim_{s\to 0}\left[\frac{1}{s^{(k_1-1)}}\cdot\frac{\frac{\Delta}{2}\left[\underset{\sim}{I}+\underset{\sim}{G}\right]_{1,2}}{\frac{\Delta}{2}\left[\underset{\sim}{I}+\underset{\sim}{G}\right]}+\frac{1}{s^{(k_2-1)}}\cdot\frac{\frac{\Delta}{2}\left[\underset{\sim}{I}+\underset{\sim}{G}\right]_{2,1}}{\frac{\Delta}{2}\left[\underset{\sim}{I}+\underset{\sim}{G}\right]}\right]$$

$$\lim_{s\to 0}\frac{\dfrac{1}{s^{(k_1-1)}}\cdot\dfrac{P_{21}}{sP_\Delta}+\dfrac{1}{s^{(k_2-1)}}\cdot\dfrac{P_{12}}{sP_\Delta}}{\dfrac{(s^2P_\Delta+P_{11})(s^2P_\Delta+P_{22})-s^2P_{12}P_{21}}{s^4P_\Delta}}=0$$

$$(k_1, k_2 \leq 3)$$

(II-42)

Then the system is steady state decoupled not only for step inputs, but also for ramp (k = 2) and parabolic (k = 3) inputs.

Returning for the special cases found in part B, the other possible plant type number matrices are:

a) $\underset{\sim}{T}_{pa} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

b) $\underset{\sim}{T}_{pb} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ or $\underset{\sim}{T}_{pb} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

For all cases, the condition (II-39) is still satisfied with the same compensator described by equations (II-4):

a) $N_{12} = 0+0 = 0$

$N_{21} = 0+0 = 0$

$M = \mathrm{Max}\{(0+2), (0+2), (0+0+4)\} = 4$

$M > N_{12}, N_{21}\,,\quad M > 0$

b) $N_{12} = 0+1 = 1$

$N_{21} = 0+1 = 1$

$M = \mathrm{Max}\{(0+1), (0+1), (0+0+0)\} = 1$

$M = N_{12} = N_{21}\,,\quad M > 0$

c) $N_{12} = 0 + 0 = 0$

$N_{21} = 0 + 0 = 0$

$$M = \text{Max}\{(0+1), (0+1), (0+0+1)\} = 1$$

$$M > N_{12}, N_{21}, \quad M > 0$$

However, in cases b and c only step inputs will be allowed, whereas in case a even a third order input can be applied.

It has been shown that the closed loop system is steady state decoupled. A number of assumptions were necessary, but the safety margin indicated by (II-42) must be enough to counterbalance some minor departure from what has been obtained to this point. On the other hand, nothing can be said about what will happen with the system response after closing the feedback path. The system may become unstable or show a poor transient response. Then in the next sections the closed loop system will be analyzed and the compensator matrix (II-41) modified to the general form

$$\underset{\sim}{G_c'} = \begin{bmatrix} g_{11}\dfrac{s+z_{11}}{s+p_{11}} & 0 \\ 0 & g_{22}\dfrac{s+z_{22}}{s+p_{22}} \end{bmatrix} \qquad \text{(II-43)}$$

so that the transient response to a given input vector can be conformed to a desired standard.

## III. THE CLOSED LOOP SYSTEM

### A. THE STEERING CONTROL

An actual steering has a certain time lag, tr between the helmsman action and the desired displacement of the rudder. The motion of the control surface begins accelerating and decelerates when reaching the final position. This (non-dimensionalized) time lag is usually taken equal to 0.1.

Figure III-1 shows the block representation of the steering control with time lag

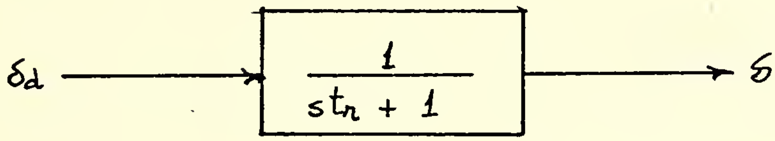


Fig. III-1.

where

$\delta_d$ is the desired rudder angle

$\delta$ is the actual rudder angle

$t_r$ is the time lag (tr = 0.1)

### B. COMPUTER SIMULATION OF THE M.I.M.O. SYSTEM

1. Modification of the Equations of Motion

If $YI_1$ and $NI_1$ are the nondimensionalized force and moment acting on ship #1 the equations of motion become

$$\frac{V_1(s)}{s}\left[a_{11}s^2+b_{11}s+c_{11}\right]+\Psi_1(s)\left[a_{21}s^2+b_{21}s+c_{21}\right]=Y_\delta\,\delta_1(s)+YI_1(s)$$

$$\frac{V_1(s)}{s}\left[a_{12}s^2+b_{12}s+c_{12}\right]+\Psi_1(s)\left[a_{22}s^2+b_{22}s+c_{22}\right]=N_\delta\,\delta_1(s)+NI_1(s)$$

$$\frac{U_1(s)}{s}\left[a_{33}s^2+b_{33}s+c_{33}\right]=0 \qquad \text{(III-1)}$$

Equations (III-1) are the same as (II-12) except for the extra terms $YI_1$ and $NI_1$.

Considering the steering control as that shown in Figure III-1 comes

$$\delta_1(s) = \frac{10\,\delta_{dL}(s)}{s+10} \tag{III-2}$$

and setting

$$IF_{11}(s) = \frac{Y_\delta \cdot \delta_{d1}(s)}{0.1s+1} + YI_1(s)$$

$$IF_{21}(s) = \frac{N_\delta\, \delta_{d1}(s)}{0.1s+1} + NI_1(s) \tag{III-3}$$

The same procedure followed for the derivation of equations (II-13) through (II-18) can be applied yielding as before

$$a_{11}\ddot{A}_1 + b_{11}\dot{A}_1 + c_{11}A_1 + a_{21}\ddot{B}_1 + b_{21}\dot{B}_1 + c_{21}B_1 = IF_{11}$$
$$a_{12}\ddot{A}_1 + b_{12}\dot{A}_1 + c_{12}A_1 + a_{22}\ddot{B}_1 + b_{22}\dot{B}_1 + c_{22}B_1 = IF_{21}$$
$$a_{33}\ddot{C}_1 + b_{33}\dot{C}_1 + c_{33}C_1 = IF_{31}$$

or with

$$I_{11} = -b_{11}\dot{A}_1 - c_{11}A_1 - b_{21}\dot{B}_1 - c_{21}B_1 + IF_{11}$$
$$I_{21} = -b_{12}\dot{A}_1 - c_{12}A_1 - b_{22}\dot{B}_1 - c_{22}B_1 + IF_{21}$$
$$I_{31} = -b_{33}\dot{C}_1 - c_{33}C_1 + IF_{31} \tag{III-5}$$

equations (III-4) can be written as

$$a_{11}\ddot{A}_1 + a_{21}\ddot{B}_1 = I_{11}$$
$$a_{12}\ddot{A}_1 + a_{22}\ddot{B}_1 = I_{21}$$
$$a_{33}\ddot{C}_1 = I_{31} \tag{III-6}$$

Solving for $\ddot{A}_1$, $\ddot{B}_1$ and $\ddot{C}_1$ one obtains

$$\ddot{A}_1 = \frac{a_{22}I_1 - a_{21}I_{21}}{a_{11}a_{22} - a_{12}a_{21}}, \quad \ddot{B}_1 = \frac{a_{11}I_{21} - a_{12}I_{11}}{a_{11}a_{22} - a_{12}a_{21}}, \quad \ddot{C}_1 = \frac{I_{31}}{a_{33}} \tag{III-7}$$

and the original variables are recovered as before,

$$V_1 = \dot{A}_1 = V_{01} + \int \ddot{A}_1\, dt$$

$$\psi_1 = B_1 = \psi_{01} + \int \dot{B}_1\, dt = \psi_{01} + \int [\dot{B}_{01} + \int \ddot{B}_1\, dt]\, dt$$

$$u_1 = \dot{C}_1 = u_{01} + \int \ddot{C}_1\, dt \tag{III-8}$$

so that in the space coordinate system

$$\dot{Y}_1 = U_1 \sin\psi_1 + V_1 \cos\psi_1$$

$$\dot{X}_1 = U_1 \cos\psi_1 - V_1 \sin\psi_1$$

$$Y_1 = Y_{01} + \int \dot{Y}_1\, dt$$

$$X_1 = X_{01} + \int \dot{X}_1\, dt \tag{III-9}$$

Since the ships are considered to be identical, equations (III-3) through (III-9) hold for both the replenishing and the receiving ships, the subscript 2 referring to the latter. Such equations were translated into DSL/360 digital computer program II.

A picewise linear approximation of forces and moments is given by the table look up and interpolation Subroutine Forces. A warning message is printed whenever the distance between the ships become less than 25 feet. For separations greater than 250 feet the ships were considered to be outside of the range of interest and the forces and moments assumed to be null. Starting with the ships exactly abeam ( $\Delta x = 0$) and observing the equilibrium conditions stated for the derivation of the equations of motion the response of the M.I.M.O. open loop system was obtained in terms of $\psi_1, \psi_2, Y_1$ and $Y_2$. The initial separation between the ships was taken as 0.2. With no controls applied, DD1 = DD2 = 0, it can be noticed from Figure III-2 and III-3 that the yaw angles diverge (bringing sterns towards each other) and the lateral separation increases as time goes by. Then a proper amount of rudder must be applied to counteract the interaction forces and moments and bring the system back to an equilibrium position.

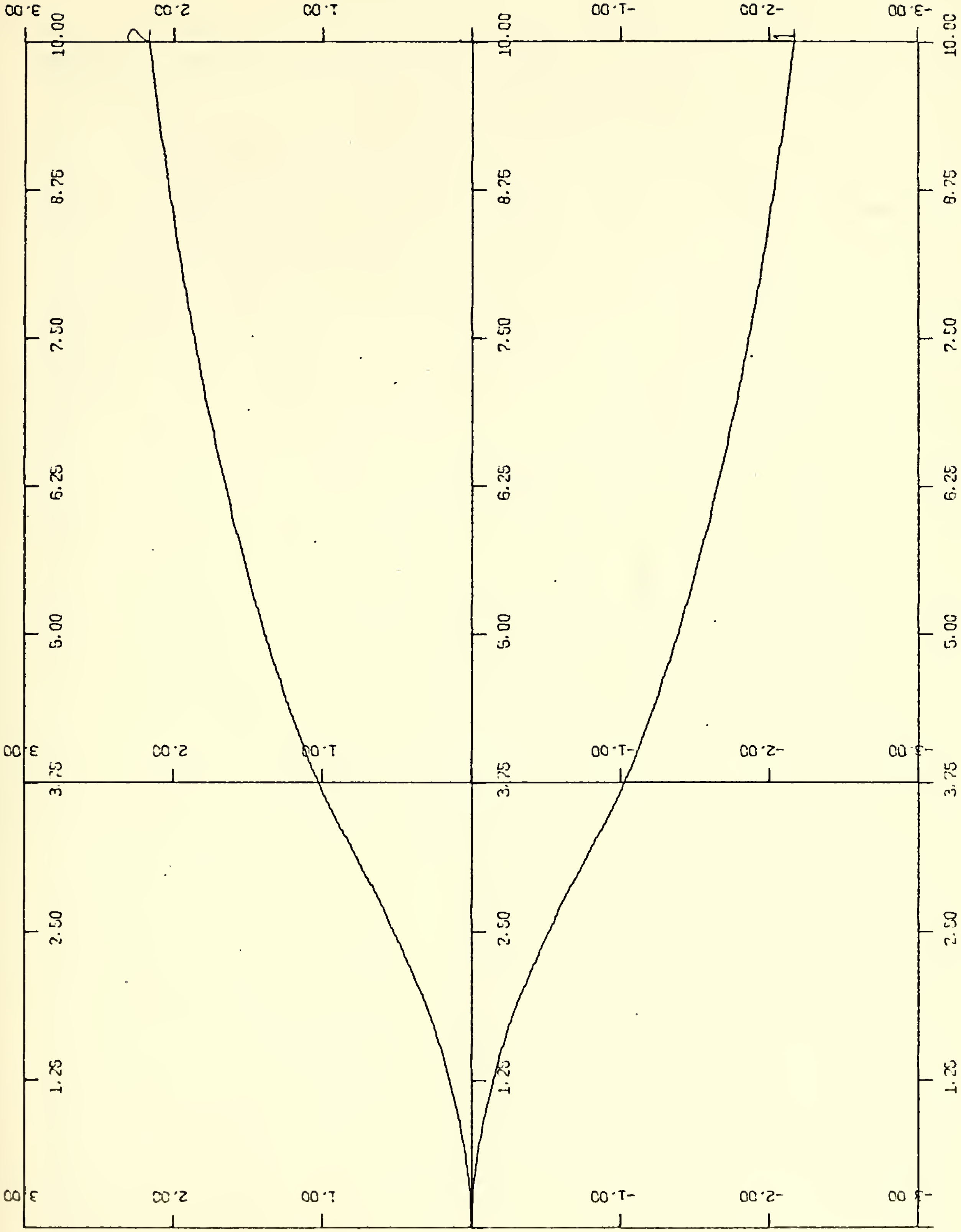


Fig. III-2. Open Loop System Response - Yaw vs. Time Y(0) = 0.2.

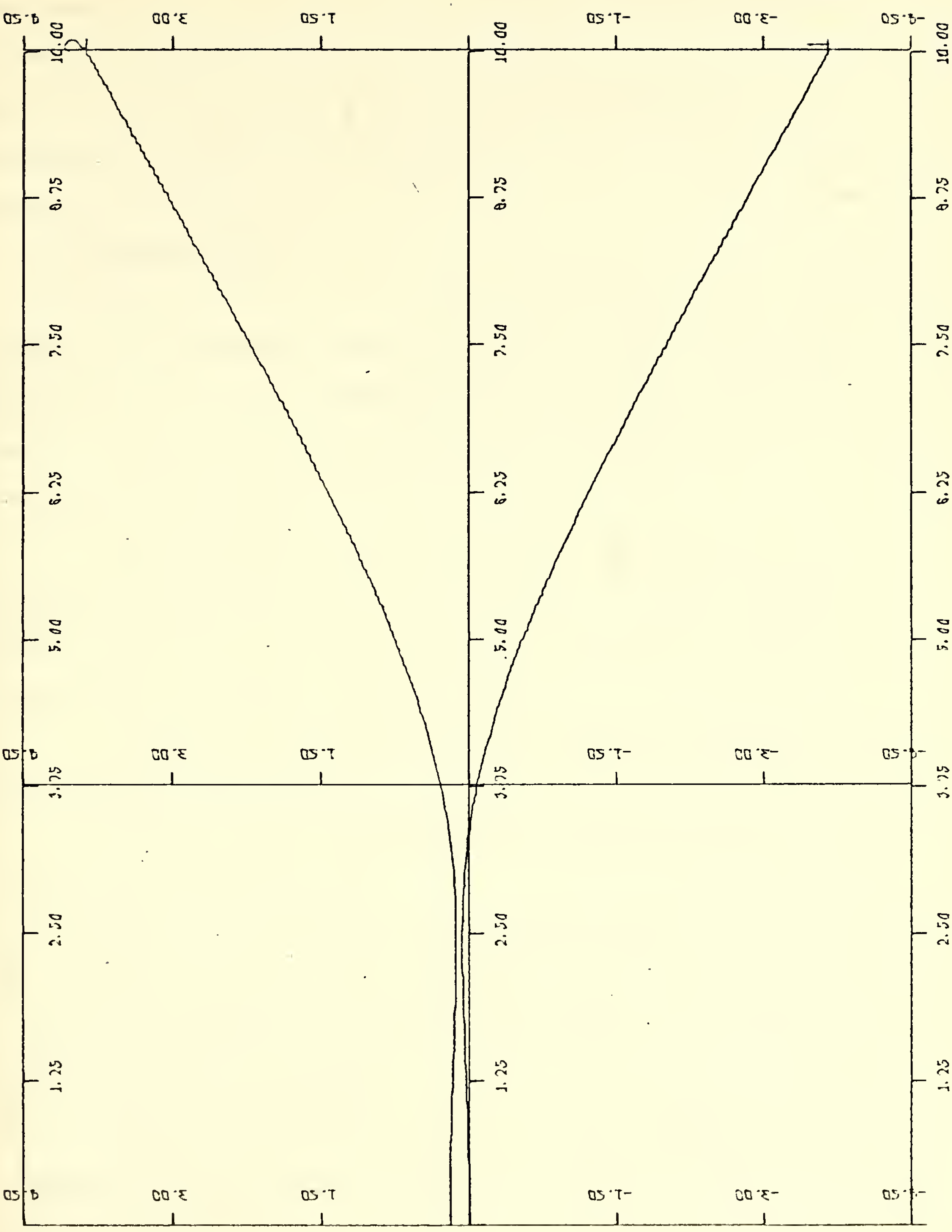

Fig. III-3. Open Loop System Response - Sway vs. Time Y(0) = 0.2.

The necessity of the control action is emphasized by analyzing Figures III-4 and III-5, obtained with initial separation 0.1. The hydrodynamics forces do not react fast enough and the interaction forces and moments causes the ships to yaw as before; the lateral separation drops to zero, indicating a collision.

2. The Control Loops

It has been shown that the closed loop system is steady state decoupled and the transient response can be made to satisfy some desired specifications by choosing adequately the parameters of a cascade compensator. The control loop will be determined to satisfy the station change problem, formulated as:

Ships #1 and #2 are originally on a straightforward motion, under equilibrium conditions; it is desired to change the lateral separation between them, keeping the original course. The general case, where both ships maneuver is shown in Figure III-6 where

$y_{o1}$ and $y_{o2}$ are the initial lateral distances from ships #1 and #2 to the space coordinate system.

$y_{d1}$ and $y_{d2}$ are the desired lateral distances from ship #1 and #2 to the space coordinate system.

The variables of interest in the control loop are the outputs $\psi_1$, $\psi_2$, their rates, the actual separation $\Delta y$ and its rate $\Delta \dot{y}$, the two latter made available by sensors devices (such as radars) that would provide continuous information.

For course keeping action the control loop should not include input yaw reference. Then one can manipulate the block diagram as shown in Figure III-7 so that with

(III-10)

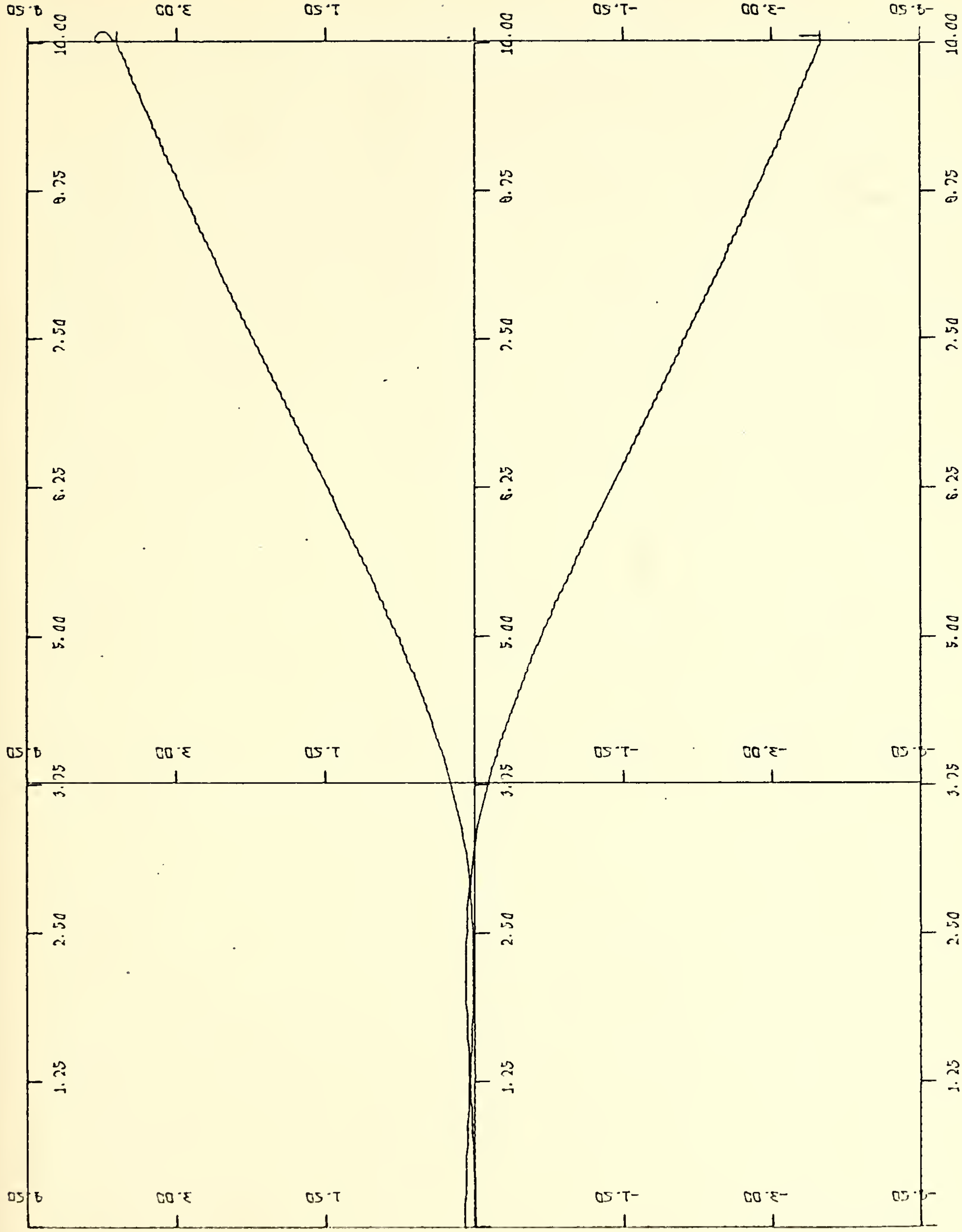


Fig. III-4. Open Loop System Response - Sway vs. Time Y(0) = 0.1

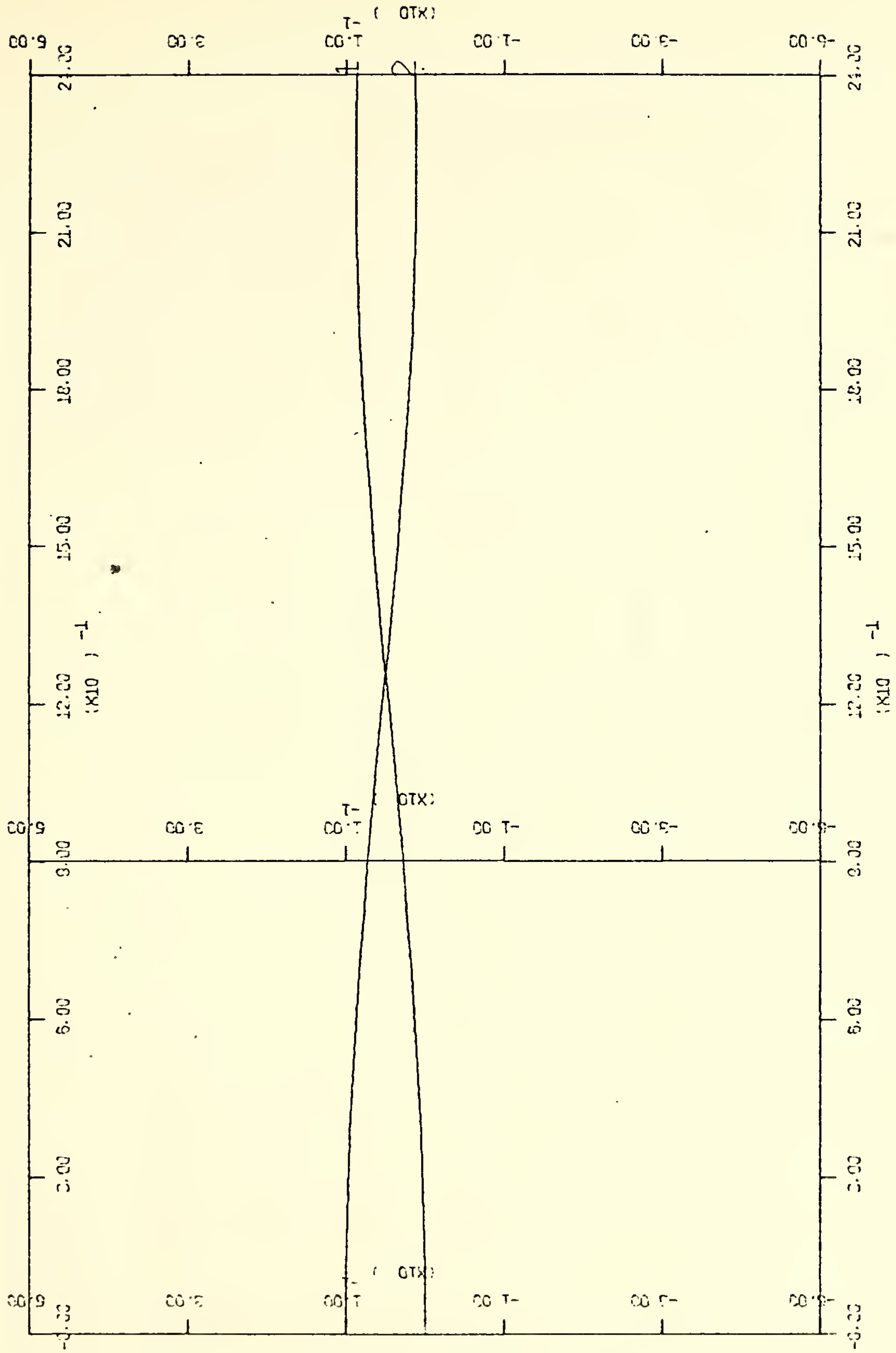

Fig. III-5. Expanded View of Fig. III-4

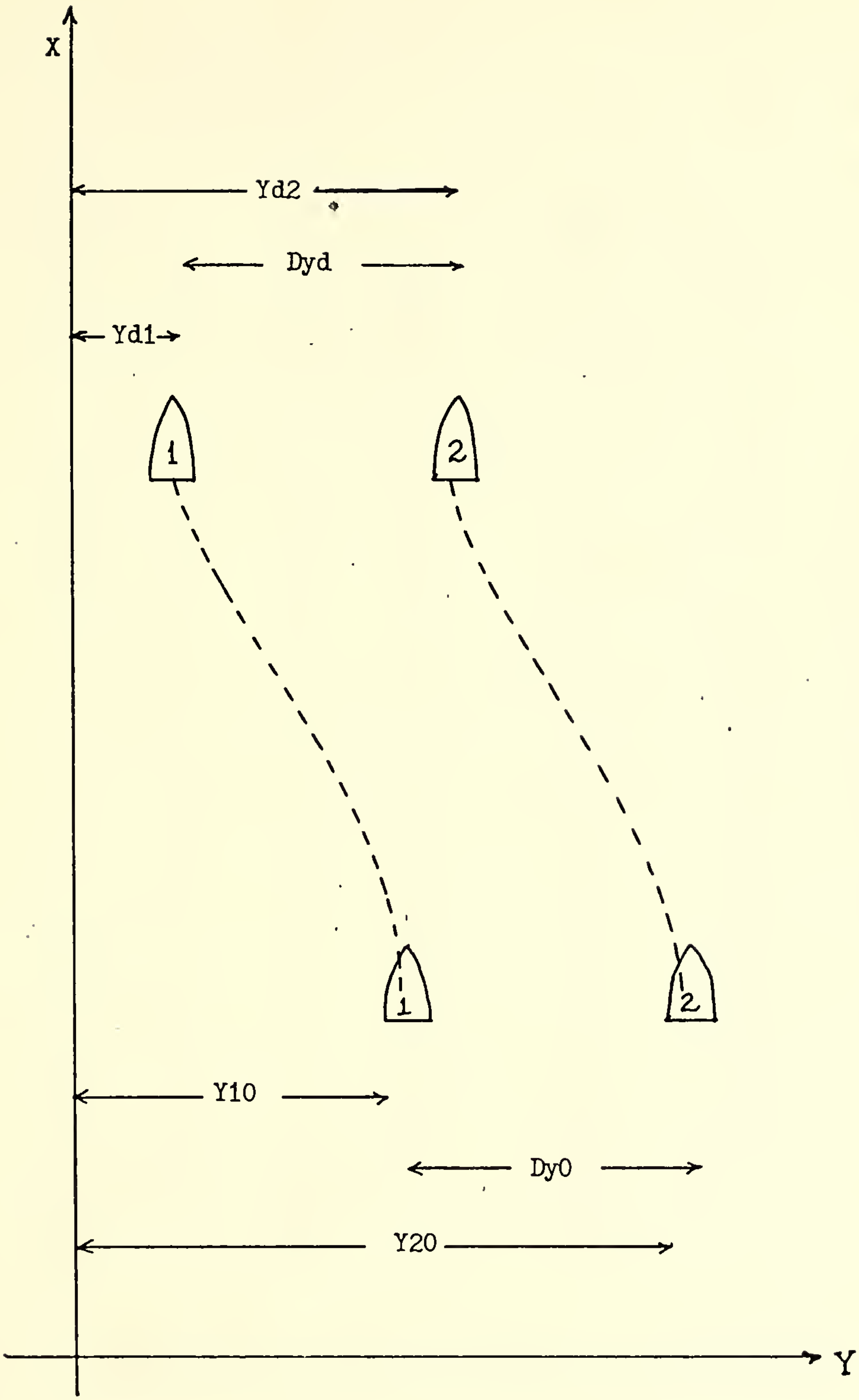


Fig. III-6. The Station Changing Problem

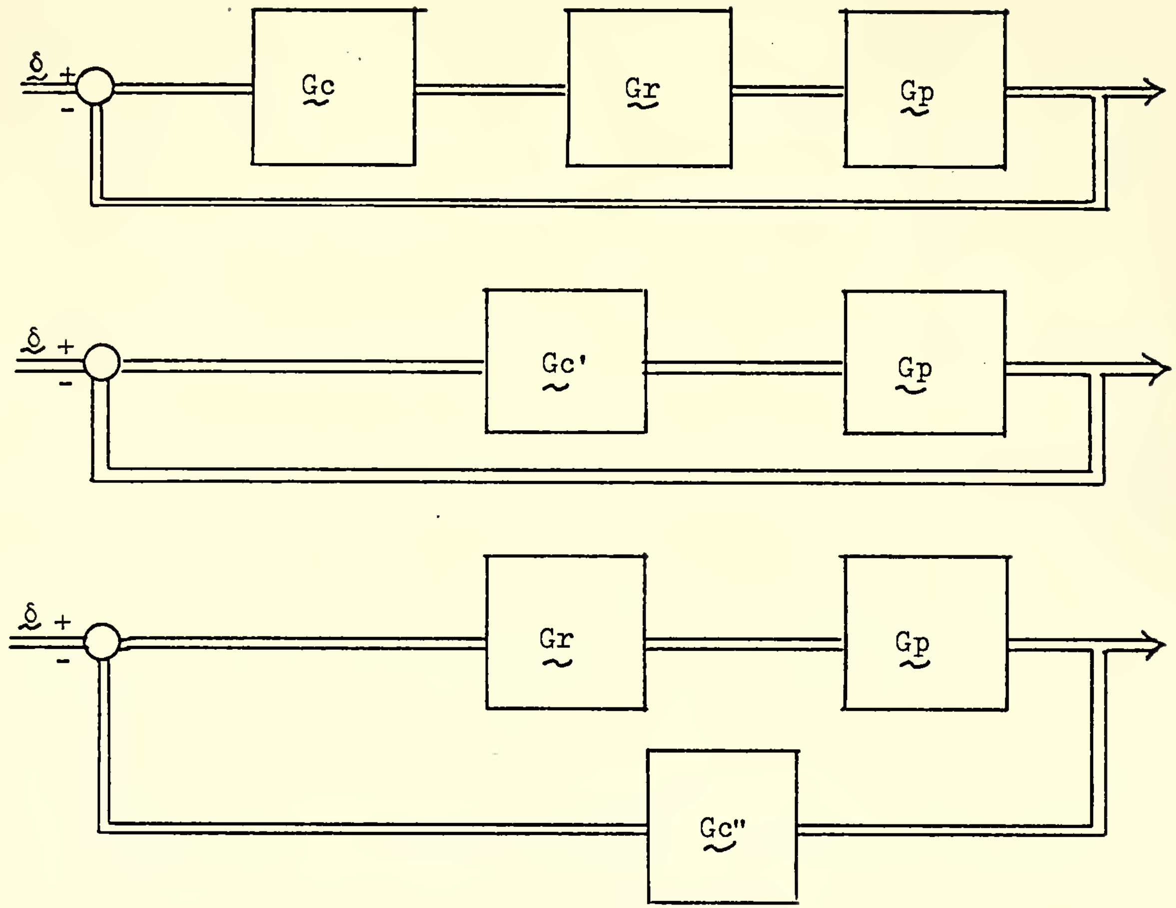


Fig. III-7. Block Diagram Manipulation

$$\underset{\sim}{G_R} = \begin{bmatrix} \dfrac{1}{t_R s+1} & 0 \\ 0 & \dfrac{1}{t_R s+1} \end{bmatrix}$$

and

$$\underset{\sim}{G_c'} = \begin{bmatrix} g_{11}\dfrac{s+z_{11}}{s+p_{11}}, & 0 \\ 0 & g_{22}\dfrac{s+z_{22}}{s+p_{22}} \end{bmatrix} = \underset{\sim}{G_R}\,\underset{\sim}{G_c}$$

(III-11)

$$\underset{\sim}{G_c''} = \begin{bmatrix} g_{11}(s+z_{11}) & 0 \\ 0 & g_{22}(s+z_{22}) \end{bmatrix} \triangleq \begin{bmatrix} K_1 + sKt_1 & 0 \\ 0 & K_2 + sKt_2 \end{bmatrix}$$

(III-12)

One degree of freedom is lost because the poles of the compensator cannot be adjusted; but the problem reduces to that of obtaining the feedback loop gains

$$K_1 = g_{11} \qquad Kt_1 = g_{11}\, z_{11}$$
$$K_2 = g_{22} \qquad Kt_2 = g_{22}\, z_{22} \tag{III-13}$$

For station changing, and then distance keeping, the control loop must contain the references $\underset{\sim}{Yd}$ and the sensor outputs. As before the problem is to obtain the feedback gains $\underset{\sim}{Kty}$ and $\underset{\sim}{Ky}$. The block diagram of the controlled plant is shown in Figure III-8, and the following relations are obtained:

$$\underset{\sim}{\delta}(s) = \underset{\sim}{G_R}(s)\,\delta_d(s)$$
$$\underset{\sim}{\delta_d}(s) = \underset{\sim}{\delta_y}(s) + (s\underset{\sim}{Kt} + \underset{\sim}{K})\underset{\sim}{\Psi}(s)$$
$$\underset{\sim}{\delta_y}(s) = \underset{\sim}{K}(\underset{\sim}{\Delta y} - \underset{\sim}{\Delta y_f}) + s\underset{\sim}{Kt}\,\underset{\sim}{\Delta y}$$

(III-14)

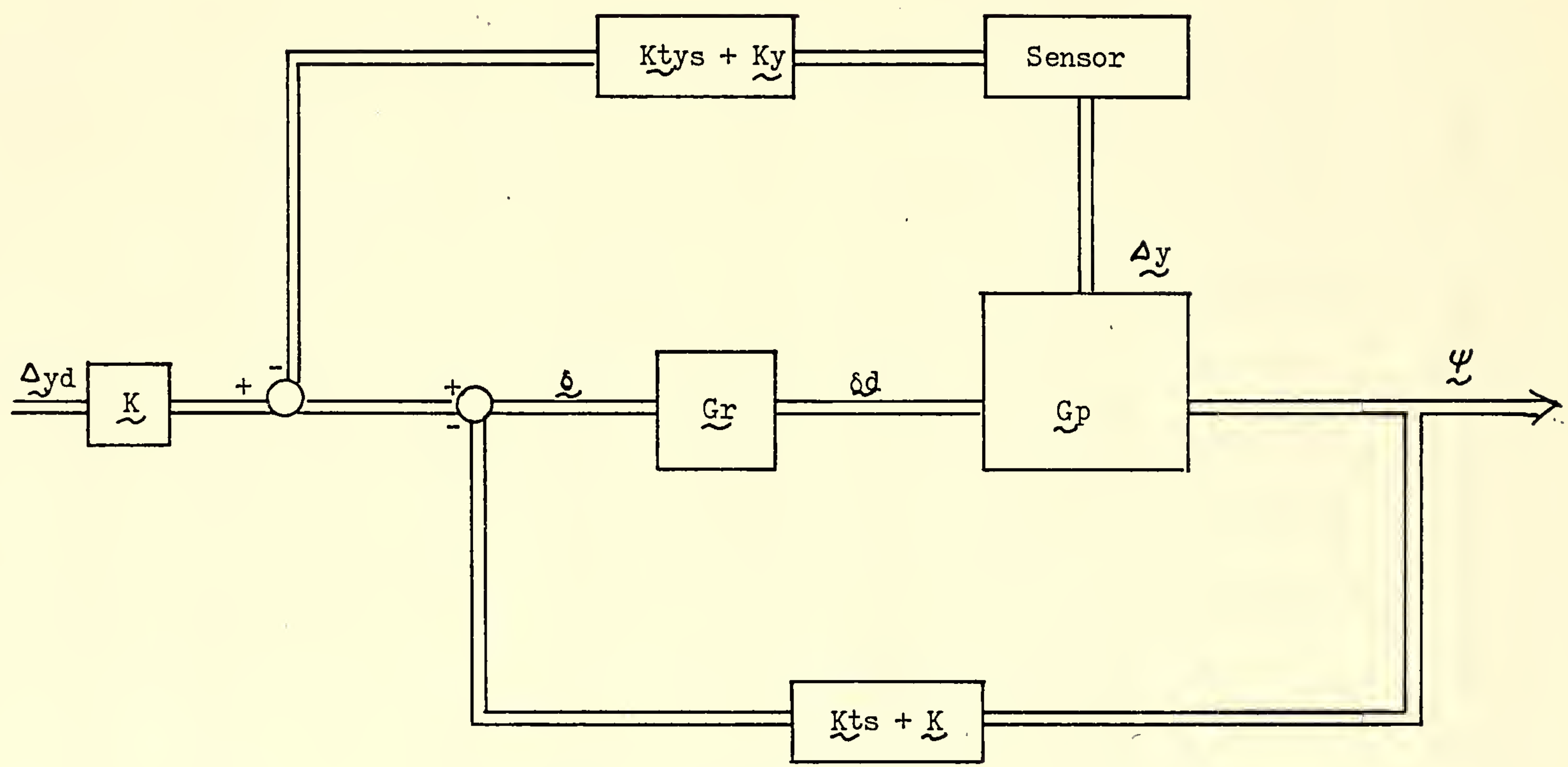


Fig. III-8. The Control Loops

then

$$\underset{\sim}{\delta}(s) = \underset{\sim}{G}_R(s)\left[(s\underset{\sim}{K}_t + \underset{\sim}{K})\underset{\sim}{\Psi} + s\underset{\sim}{K}_t\underset{\sim}{\Delta y} + \underset{\sim}{K}(\underset{\sim}{\Delta y} - \underset{\sim}{y}_f)\right] \tag{III-15}$$

where

$$\underset{\sim}{\Delta y}_f = \begin{bmatrix} -(y_{2f} - y_{1f}) \\ (y_{2f} - y_{1f}) \end{bmatrix} = \begin{bmatrix} -y_{2f} \\ y_{2f} \end{bmatrix}$$

$$\underset{\sim}{\Delta y} = \begin{bmatrix} -(y_2 - y_1) \\ (y_2 - y_1) \end{bmatrix} \triangleq \begin{bmatrix} -\Delta y \\ \Delta y \end{bmatrix} \tag{III-16}$$

expanding (III-15) gives

$$\delta_1(s) = \frac{1}{t_n s + 1}\left[(sK_{t_1} + K_1)\Psi_1 - sK_{ty_1}\Delta y - K_{y_1}(\Delta y - y_{2f})\right]$$

$$\delta_2(s) = \frac{1}{t_n s + 1}\left[(sK_{t_2} + K_2)\Psi_2 + sK_{ty_2}\Delta y + K_{y_2}(\Delta y - y_{2f})\right] \tag{III-17}$$

Computer program II is therefore modified to give Program III where equations (III-3) are now

$$IF_{11} = \frac{Y_\delta}{0.1s+1}\left[(sK_{t_1} + K_1)\Psi_1 - sK_{ty_1}\Delta y - K_{y_1}(\Delta y - y_{2f})\right] + YI_1(s)$$

$$IF_{21} = \frac{N_\delta}{0.1s+1}\left[(sK_{t_1} + K_1)\Psi_1 - sK_{ty_1}\Delta y - K_{y_1}(\Delta y - y_{2f})\right] + NI_1(s)$$

$$IF_{31} = -\frac{X_u}{s} \tag{III-18}$$

whereas for ship #2

$$IF_{12} = \frac{Y_\delta}{0.1s+1}\left[(sK_{t_2} + K_2)\Psi_2 + sK_{ty_2}\Delta y + K_{y_2}(\Delta y - y_{2f})\right] - YI_2(s)$$

$$IF_{22} = \frac{N_\delta}{0.1s+1}\left[(sK_{t_2} + K_2)\Psi_2 + sK_{ty_2}\Delta y + K_{y_2}(\Delta y - y_{2f})\right] - NI_2(s)$$

$$IF_{32} = -\frac{X_u}{s} \tag{III-19}$$

and equations (III-4) through (III-9) hold so that no other changes are required for simulation.

Computer program II is therefore modified to include equations (III-18) and (III-19); the values of the feedback loop gains are calculated as described in the next section.

## IV. PARAMETER OPTIMIZATION

The values of $K_1$, $K_2$, $Kt_1$, $Kt_2$, $Ky_1$, $Ky_2$, $Kty_1$, $Kty_2$ that would cause the system to follow an assigned behavior with minimum possible deviation are hardly expected to be obtained using classical methods of design of feedback systems because of the empirical and nonlinear form of some quantities involved in the problem. Then a computer-aided technique for parameter optimization will be used.

### A. THE COST FUNCTION

The optimal set of feedback loop gains can be defined [8] as the one which causes the system

$$\dot{\underset{\sim}{y}}(t) = \underset{\sim}{C}(\underset{\sim}{u},\underset{\sim}{v},\underset{\sim}{\psi})\,\underset{\sim}{F}(\underset{\sim}{u},\underset{\sim}{v},\underset{\sim}{\psi},\underset{\sim}{YI},\underset{\sim}{NI})\,\underset{\sim}{\delta}(t)$$

to follow a trajectory Y* that minimizes the cost function (also called performance measurement)

$$J = \underset{\sim}{h}[\underset{\sim}{y}(t_f), t_f] + \int_{t_0}^{t_f} \underset{\sim}{g}[\underset{\sim}{y}(t), \underset{\sim}{u}(t), t]\,dt$$

where (IV-1)

$\underset{\sim}{u}(t)$ denotes the forcing function matrix

$t_0, t_f$ the initial and final time of the problem

The form of the functions $\underset{\sim}{h}$ and $\underset{\sim}{g}$ depends on what one desires to minimize. For the system and problem under consideration, the lateral distance between the two ships, maneuvering at close proximity, is of capital importance; a safe underway replenishment (UNREP) operation requires a rather accurate station keeping, and it is clear that a reasonable separation between the ships must be observed to avoid risks of collision. In today's practiced Naval tactics the replenishing ship is only responsible for

course keeping, whereas the receiving ship is responsible for both course and station keeping. Then if the ideal path of the replenishing ship is assigned to be the X-axis of the space coordinate system ($y_{1d} = 0$) and if one desires to bring the receiving ship close to that axis by a distance $y_{2d}$, following a desired path, the form of the cost function J in (III-20) can be taken as

$$J = [\underset{\sim}{y}(t_f) - \underset{\sim}{y}(d)]^T \underset{\sim}{H} [\underset{\sim}{y}(t_f) - \underset{\sim}{y}_d] + \int_{t_0}^{t_f} [\underset{\sim}{C}(t) - \underset{\sim}{C}_i(t)]^T \underset{\sim}{Q} [\underset{\sim}{C}(t) - \underset{\sim}{C}_i(t)] dt \qquad \text{(IV-2)}$$

where

$\underset{\sim}{H}$, $\underset{\sim}{Q}$ are real symmetric positive semi-definite weighting matrices.

$\underset{\sim}{C}(t)$, $\underset{\sim}{Ci}(t)$ are respectively the actual and the desired trajectories.

$t_f$ denotes the final time of evaluation and it should be longer than the system settling time, so that steady state accuracy is attained.

The format of the cost function J given by equation (III-21) is adequate to the problem. The values of $\underset{\sim}{H}$, $\underset{\sim}{Q}$, $\underset{\sim}{Ci}$ and $t_f$ must be conveniently selected by the designer so that the minimum value obtained will actually indicate that the set of parameters used in the calculation will make the system respond in the desired way. Specific rules cannot be established and the designer is left the task of determining some suitable numerical values to be used in (IV-2), and then check the cost function by analyzing the system response obtained when using the optimum parameters. The confrontation will result in the parallel outcome of a realistic expression for J as well as a particular optimal trajectory and the corresponding parameters.

Some simplifications can be applied in the initialization of the problem, provided that reasonable justifications are pertinent:

a) The deviation of the lateral distances $y_1$ and $y_2$ from the desired distance $y_{1d}$ and $y_{2d}$, i.e., the terms $\Delta y_1 = y_1 - y_{1d}$ and $\Delta y_2 = y_2 - y_{2d}$

must be minimized for all time greater than the time required by the maneuver, and not only for some assumed final time $t_f$; it can even happen that at $t = t_f$ either $y_1 - y_{1d}$ or $y_2 - y_{2d}$ (or both) are passing through a zero value but during an unstable oscillation or along a divergent trajectory, so that the ships will never be in the desired situation. Then one can take

$$\underset{\sim}{H} = \underset{\sim}{O}$$

b) As a first approximation, it is usual to make $\underset{\sim}{Q}$ a diagonal matrix. This assumption leads to a less complicated expression for the integrand of (IV-2), which is very desirable for long-hand operations. For the case under consideration such simplification is not the main point; the integration to be performed will involve non-linear functions of several variables, hardly performed by analytical means - but readily evaluated by numerical methods, using a digital computer. Even so the diagonalization of $\underset{\sim}{Q}$ remains desirable because it will be much simpler to adjust the two non-zero terms when investigating the most suitable cost function concerning the problem. Moreover, this classic assumption is usual because if $C_1 - C_{1i}$ and $C_2 - C_{2i}$ both have small values, as a result of the minimization procedure, their product will also be small.

c) The choice of the non-zero entries of $\underset{\sim}{Q}$ turns out to be the crucial part of the problem, since the assignment of the ideal trajectories, in spite of being restricted by the ship's dynamics capabilities, is somewhat free.

If the expressions for $\underset{\sim}{C} - \underset{\sim}{Ci}$ involves variables concerning quantities of unequal dimension (such as angles and distances) or like dimensioned but with different meaning in the problem (such as distances parallel to the x and y axes) the terms of $\underset{\sim}{Q}$ must take that into account, having

embedded conversion and/or weighting factors which reflect how harmful the unity error of one variable is when compared with another.

As stated before, a safe distance between the ships is the major concern in replenishment at sea operation; since the ships forward velocities will not change, a negligible variation of the distance along the X-axis is expected and this variable shall not be included in $\underset{\sim}{C}$ and $\underset{\sim}{Ci}$. Its value must be calculated during the maneuver, to warn the designer for the necessity of taking it into consideration if it exceeds a reasonable value. A similar argument holds for excluding the yaw angles. With the ships moving as desired, in paths parallel to the X-axis and keeping the assigned separation, these variables will be meaningless in the cost function.

A constraint to be imposed during the calculations or verified after obtaining a solution, is that applied rudder angles do not exceed the maximum allowed deflections and that the maneuver does not require sudden changes in the ships headings.

The above discussion leaves distances to the X-axis as the remaining variable of interest, the simplest possible situation for investigating adequate values for the diagonal of $\underset{\sim}{Q}$.

The expression of the cost function J reduces to

$$J = \int_{t_0}^{t_f} \left[ q_{11} (c_1 - c_{i1})^2 + q_{22} (c_2 - c_{i2})^2 \right] dt \tag{IV-4}$$

B. THE IDEAL RESPONSE

The replenishing ship, as aforementioned, is assigned the path along the space coordinate system x-axis. Then its ideal response is

$$c_{i1}(t) \triangleq y_1(t) = 0 \ , \quad \forall t \in [t_0, t_f]$$

The receiving ship is assigned an ideal response defined by the step response that a ship with identical characteristics would have if no interaction forces and moments were present and been controlled by a distance keeping loop in which the feedback gains would cause the response to be critically damped, $\zeta = 1$. Appendix B contains the calculation of the parameters involved in such idealized response, shown in Figure B-2 of that Appendix.

The assignments just done are both feasible, considering the ship's dynamics and the compensation scheme, realistic and adequate for the proposed way of solving the design problem:

a) The settling time of the ideal trajectory assigned to the receiving ship obtained from Figure A-II-2 ($t_{si} = 40$ sec) gives an idea of the minimum time required by the maneuver; an extension of the order of 50% is enough to evaluate the actual system performance and therefore one can take the final time for calculations as $t_f = 60$ sec.

b) That ideal trajectory can be obtained in the same way as the actual ones, i.e., integrating equations of motion. The alternative would be to define it by a set of points and then to use a table look up (y vs. time) and interpolation device, with evident disadvantages of inaccuracy.

Equation (IV-4) is written as

$$J = \int_{t_0}^{t_f} \left[ q_{11} y_1^2 + q_{22} (y_2 - y_{i2})^2 \right] dt \tag{IV-5}$$

The weighting factors $q_{11}$ and $q_{12}$ are the only parameters to be chosen.

Recalling that $y_{i2}$ is obtained with no forces and moments present, it follows that this trajectory is referred to a fixed line, namely the X-axis. The actual situation requires the leading ship to establish the tracking ship reference path so that the latter can perform her chores of station keeping and following the other's course. Then it is very

important that the leading ship quickly assumes a steady course and does not deviate from her path. This necessity is stressed in (IV-5) taking $q_{11}$ much greater than $q_{22}$.

A well designed pathkeeping loop will allow the term $y_1^2$ to contribute for increasing the cost function only in a short phase of the maneuver, dropping to an essentially zero value ever after, where J would depend mostly on the deviations of the tracking ship from the corresponding ideal response. The term $q_{22}(y_2 - y_{i2})^2$ may become negligible compared with $q_{11}y_1^2$ in the first seconds, but if its minimization does not require excessive control action or becomes too much time consuming (which could be noticed by the observation of a large error at the end of the interval) relatively large deviations of the tracking ship can be tolerated.

After some trials using the simulation described in the next section, the ratio

$$\frac{q_{11}}{q_{22}} = 10$$

was selected. The final expression for J is therefore

$$J = \int_0^{60} \left[ 10 y_1^2 + (y_2 - y_{i2})^2 \right] dt \tag{IV-6}$$

## V. COMPUTER AIDED DESIGN

As stated in previous sections, to calculate the feedback loop gains using classical methods is questionable for the case being studied. The approach introduced in Section IV is suitable for a digital computer aided design, where a systematic procedure will eventually lead to the values of the adjustable parameters which minimize the cost function J, and therefore make the system follow the assigned trajectories.

Equations (III-4) through (III-9), (III-12) and (III-19) are those required for the ship's dynamics simulation and for obtaining the ideal response for the trailing ship. Equation (IV-6) gives the value of the cost function.

As mentioned in Section III-2, a simple modification in DSL/360 computer program II is required to simulate the closed loop system response (provided that the feedback gains have been found), so that a graphical representation is readily obtained. A subroutine for calculating the optimal parameters could be called by the INITIAL region of the DSL program but experience has shown that a closer control and tracking of the intermediate steps of the solution are desirable; then it becomes more suitable to write an independent program for obtaining the optimal values and use them as input data to the simulation.

### A. THE MINIMIZATION PROGRAM CP-III

Subroutine BOXPLX [6,9] (constrained minimization of multivariable functions by the complex method of J.M. Box) was used to calculate the set of optimal feedback loop gains. The main data required, which must be supplied by the calling program are:

a) Number of variables - 8 ($K_1$, $Kt_1$, $K_2$, $Kt_2$, $Ky_1$, $Kty_1$, $Ky_2$, $Kty_2$);

b) Number of auxiliary variables - zero. Auxiliary variables are used for implicit constraints; one could, for example, express the rudder deflection as a function of the feedback loop gains and use its value as an implicit constraint that would eliminate a set of parameters whenever exceeded. However the amount of extra core and CPU time that would be necessary indicates that the usual engineering approach (finding the result and then check for any implicit constraint violation) is advisable here.

c) Number of trials - for each set of variables generated by BOXPLX in its search for the minimum value of the cost function, the equations of motion of the two ships and that relative to the ideal response are integrated. This assertive suffices to indicate that the higher the allowed number of trials, the longer will be the run time. On the other hand, after exceeding that number, BOXPLX gives the best minimum encountered, which will not necessarily be the absolute minimum of J. The approach used consisted of making a couple of runs with a limited number of trials (30) and then using the intermediate results for adjusting the bounds and starting values (as described in the next items) and requesting a long run in which 2,000 trials were allowed.

d) Lower bounds - To avoid the possibility of positive feedback, the initial values for the lower bounds were set to zero.

e) Upper bounds - The wider the permitted range in which the variables are allowed to vary, the more difficult becomes the task of locating the minimum, even limiting the number of trials to 2,000. The upper bounds were initially set to 2 in the short runs and then, together

with the lower bounds, adjusted to centralize the band about the best found optimal values.

f) Starting values - In the first run the starting values were all set to zero so that the value of the cost function for the uncompensated system was obtained (See Table V-2) to serve as reference for other values eventually found. For the next runs the starting values were taken as the best optimal values obtained in the previous run.

The BOXPLX user must also supply a function subprogram FE where the object function is evaluated (See next item) and another KE where the implicit constraints are tested.

1. Evaluation of the Cost Function

The centralized integration process used in DSL will not allow the use of this language in the re-entrant way used by subroutine BOXPLX. Then the system had to be simulated using fortran statements, the differential equations being solved by the fourth order Runge-Kutta method (function RKLDEQ of the IBM Scientific Subroutine Package).

The differential equations were written in state variable form and the correspondence between the symbols previously adopted and those then introduced is shown in the Table V-1.

The Fortran program translates equations (III-4) through (III-9), (III-18), (III-19) and (IV-6). The initial conditions are all zero, except the distances from the receiving and the "ideal" ships to the x-axis, y(10) and y(15). The final desired separation is defined by the variable DFIN.

The cost function $J = y(20)$ is evaluated for a time interval of 60 seconds, in steps of 0.3 seconds and has its value returned to BOXPLX.

TABLE V-1

STATE VARIABLES SYMBOLS

| VARIABLE | DSL SYMBOL | STATE VARIABLE |
|---|---|---|
| $v_1$ | ADOT1 | Y(1) |
| $\dot{v}_1$ | ADDOT1 | YDOT(1) |
| $\psi_1$ | B1 | Y(2) |
| $\dot{\psi}_1$ | BDOT1 | Y(3) |
| $\ddot{\psi}_1$ | BDDOT | YDOT(3) |
| $u_1$ | CDOT1 | Y(4) |
| $\dot{u}_1$ | CDDOT1 | YDOT(4) |
| $y_1$ | Y1 | Y(5) |
| $\dot{y}_1$ | YDOT1 | YDOT(5) |
| $v_2$ | ADOT2 | Y(6) |
| $\dot{v}_2$ | ADDOT2 | YDOT(6) |
| $\psi_2$ | B2 | Y(7) |
| $\dot{\psi}_2$ | BDOT2 | Y(8) |
| $\ddot{\psi}_2$ | BDDOT2 | YDOT(8) |
| $u_2$ | CDOT2 | Y(9) |
| $\dot{u}_2$ | CDDOT2 | YDOT(9) |
| $y_2$ | Y2 | Y(10) |
| $\dot{y}_2$ | YDOT2 | YDOT(10) |
| $x_1$ | X1 | Y(17) |
| $\dot{x}_1$ | XDOT1 | YDOT(17) |
| $x_2$ | X2 | Y(18) |
| $\dot{x}_2$ | XDOT2 | YDOT(18) |
| $v_i$ | ------ | Y(11) |
| $\dot{v}_i$ | ------ | YDOT(11) |

TABLE V-1 cont'd.

| VARIABLE | DSL SYMBOL | STATE VARIABLE |
|---|---|---|
| $\psi_i$ | ------- | Y(12) |
| $\dot{\psi}_i$ | ------- | Y(13) |
| $\ddot{\psi}_i$ | ------- | YDOT(13) |
| $u_i$ | ------- | Y(14) |
| $\dot{u}_i$ | ------- | YDOT(14) |
| $y_i$ | ------- | Y(15) |
| $\dot{y}_i$ | ------- | YDOT(15) |
| $x_i$ | ------- | Y(19) |
| $\dot{x}_i$ | ------- | YDOT(19) |
| J | ------- | Y(20) |

## B. RESULTS

Table V-2 summarizes the results obtained using the minimization program CP-III. It can be noticed that the cost function drops from 161.434 when the system is uncompensated to a few tenths when the optimal parameters are used. Since the problem starts with the ships abeam and all other initial conditions (inclusive forces and moments) equal to zero, these interactive perturbations will not increase gradually as it would happen if the trailing ship was approaching the leading ship, but will immediately rise to values corresponding to the initial separation between the ships, and then vary as that separation changes. For this reason the optional parameters are different in each of the studied cases; anyway it can be observed that each parameter has a definite order of magnitude.

### 1. The System Response

The graphical representation of the system response, for the same conditions and parameters indicated in Table V-2, was obtained with DSL program CP-IV, in terms of the following variables.

#### a) Sway

In all cases similar responses are observed. The leading ship initially overshoots and then resumes the original course over a parallel path, with an error of the order of 10 ft. Its compensators, like those of the trailing ship, actuate in opposite directions when the problem begins; the course keeping loop decreases the control action that will bring the ships close together. The net results show a kind of symmetry. The leading ship, for a short period, is dominantly actuated by the distance keeping loop but in about seven seconds it is the course keeping loop that makes the ship follow the desired trajectory. With the tracking ship the situation is reversed. The course control loop dominates

TABLE V-2

OPTIMAL FEEDBACK LOOP GAINS

| RUN | $Y_2$ at t = 0 | yd | k1 | kt1 | k2 | kt2 | ky1 | kty1 | ky2 | kty2 | $J_{min}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.40 | 0.30 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 161.434 |
| 1 | 0.40 | 0.30 | 2.843 | 2.845 | 2.953 | 1.984 | 2.975 | 2.727 | 1.506 | 2.812 | 0.103 |
| 2 | 0.36 | 0.30 | 3.069 | 3.080 | 2.853 | 2.141 | 3.320 | 2.555 | 1.533 | 2.608 | 0.307 |
| 3 | 0.40 | 0.24 | 2.975 | 2 775 | 2.917 | 2.042 | 3.008 | 2.783 | 1.511 | 2.878 | 0.267 |

Note: Adimentional distances tabulated; convert to actual distances multiplying by the ship's length L = 528 ft.

initially; after three seconds the distance keeping loop becomes more effective and the ship is brought to the assigned position, when the loop actions agree.

Figures V-1, V-5 and V-11 show the sway vs. time response for each of the three runs.

b) Yaw

The yaw angles are shown in Figures V-2, V-6 and V-12. Those referring to the leading ship are of smaller amplitudes. In steady state their values are equal and of opposite signs, a consequence of the assumption that the lateral forces and moments acting in each ship are also equal and of opposite signs. This result agrees with Reference 4.

c) Distance Between the Ships

The longitudinal distance between the ships is essentially zero for all time and for any initial situation. This result indicates that nothing was lost in neglecting this term when selecting the cost function.

The lateral distances curves should be close to the assigned ideal response, Figure B-2. As can be observed, a good performance is obtained, since steady state constancy is attained and the relatively small errors are always positive, i.e., the ships do not come closer than the desired final separation, meeting a necessary safety requirement. Figures V-3, V07 and V-13 show how the lateral and the longitudinal distances vary with time.

d) Geographic Displacement

It was seen that the longitudinal separation is zero. Then a geographic plot (sway vs. surge) of the ships trajectories can be obtained taking the motion of one of the ships along the X-direction as

reference. The results are shown in Figures V-4, V-8 and V-14 and the resemblances with the curves of sway vs. time are evident, so that no other comments seem to be necessary.

e) Rudders-Deflections

It was required that the rudder deflections must not exceed the allowed maximum values.[2] For the studied cases, (all of them concerning small displacements realized in a reasonable time), the control action was kept well below those levels. In steady state, the rudder angles applied to each ship are equal and of opposite signs, in order to compensate the constant interaction forces and moments.

Figures V-9 and V-15 show the rudder history of the leading ship and Figures V-10 and V-16 the same for the tracking ship.

Figures V-16 and V-17 show the individual contribution of each feedback loop in the resultant control effort. The vertical scale comprises the allowed range of permitted rudder angles. The results agree with the analysis performed till then and attend the requirements outlined in Section I-B.

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[2]For the Mariner, the excursions of the rudder are limited to $\pm 30^{\circ}$ (approximately 0.5 radians)[2].

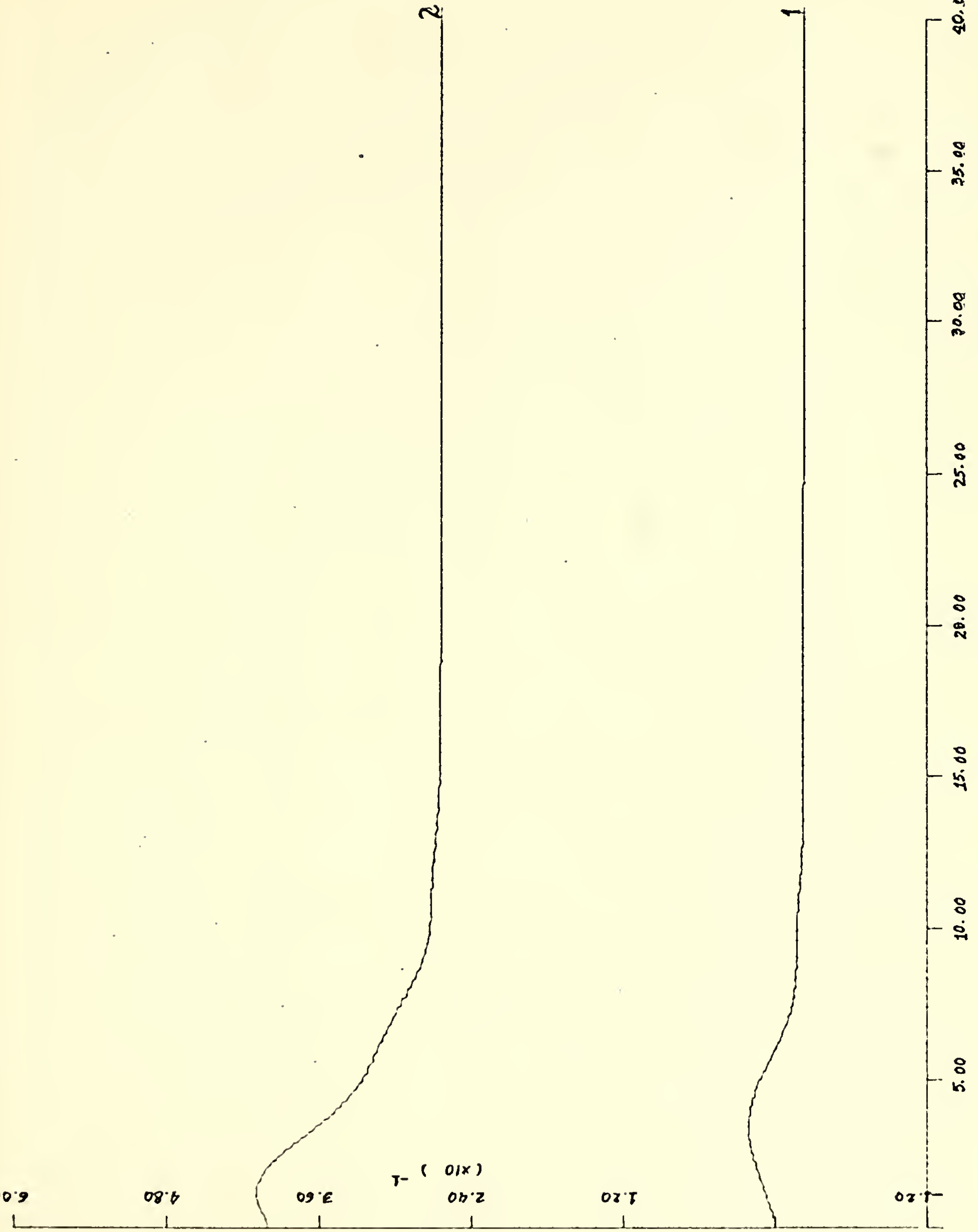



Fig. V-1. Sway vs. Time Y(0) = 0.4, YD = 0.3

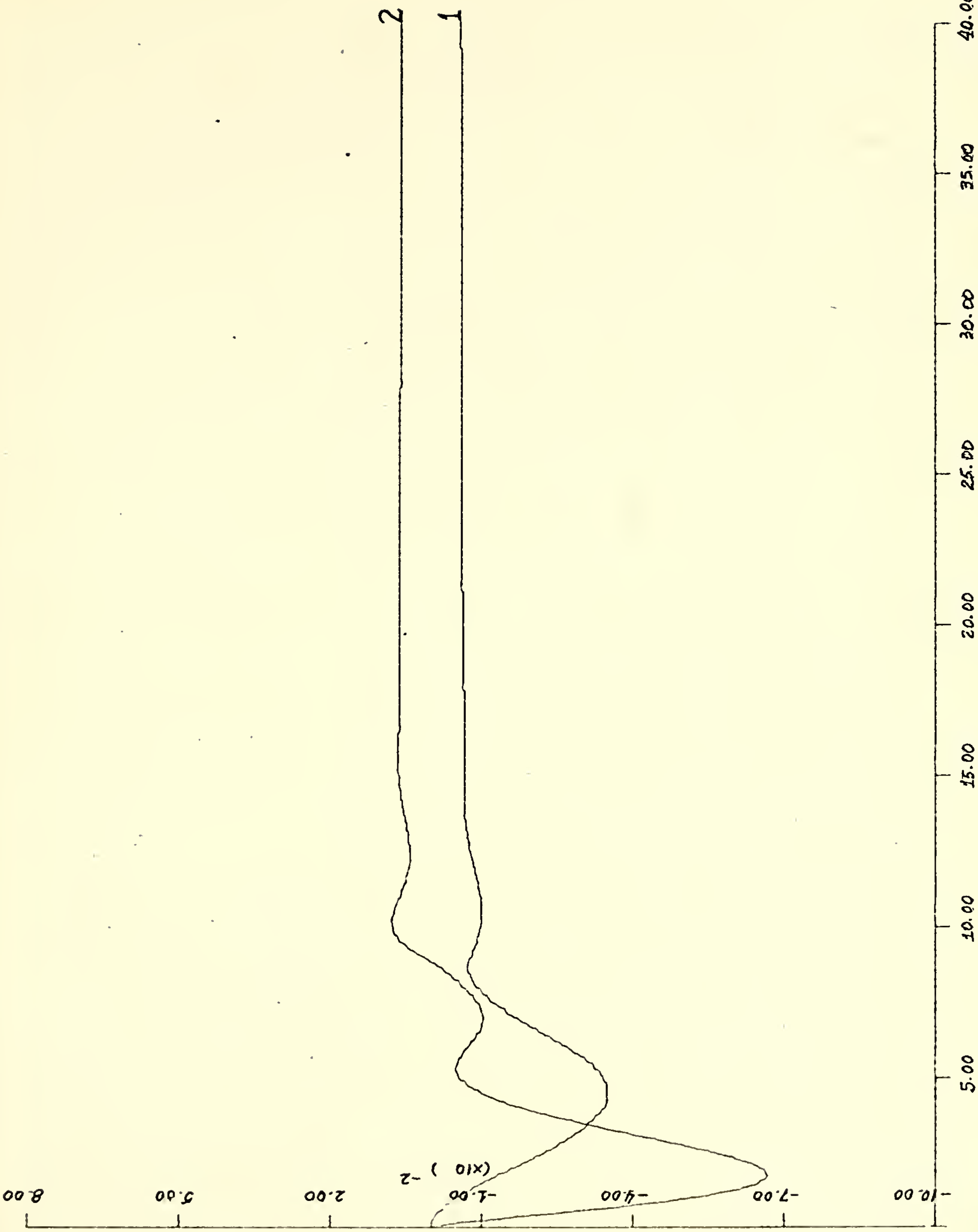



Fig. V-2. Yaw vs. Time Y(0) = 0.4, YD = 0.3

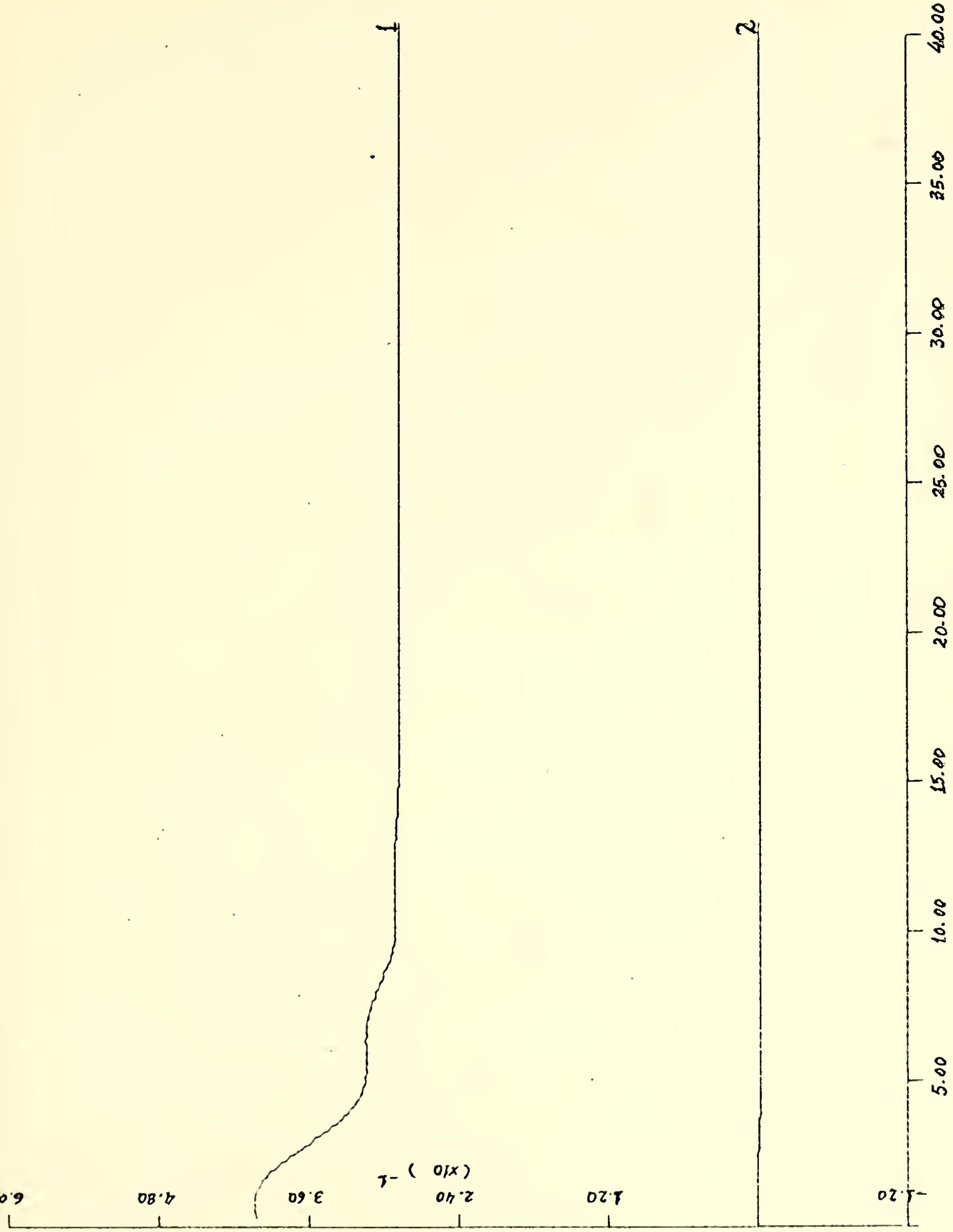



Fig. V-3. Transverse and Longitudinal Separations vs. Time

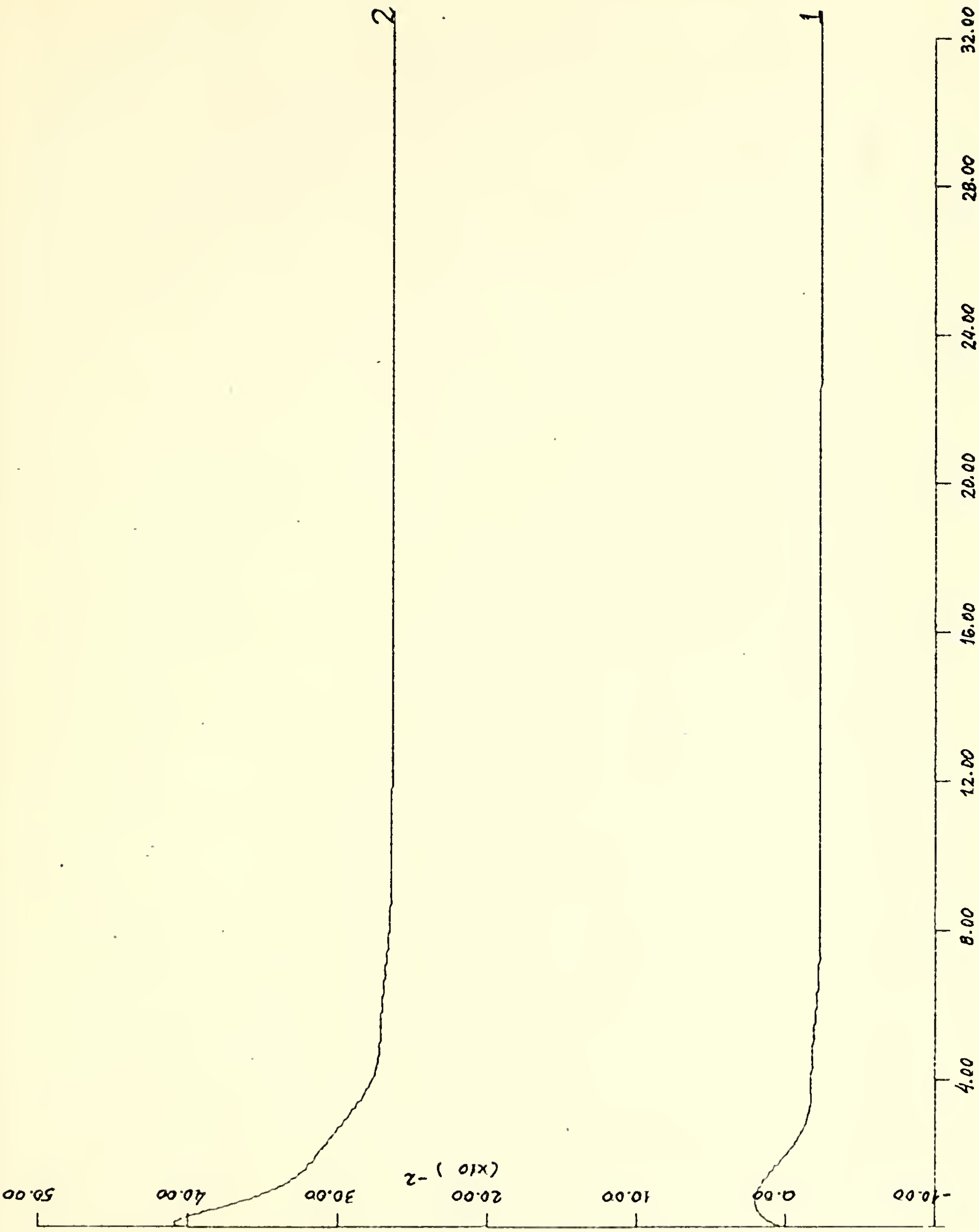



Fig. V-4. Sway vs. Surge Y(0) = 0.4, YD = 0.3

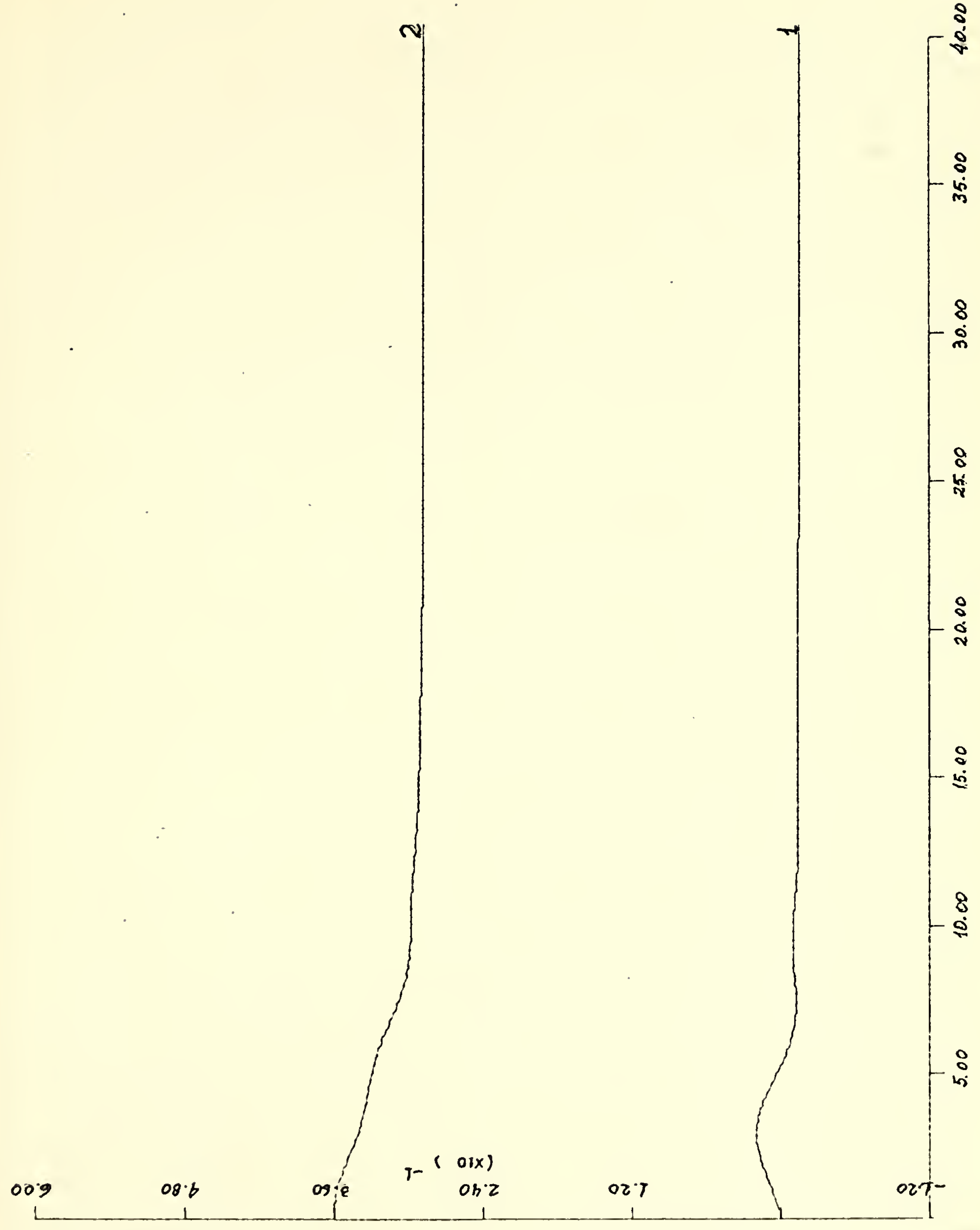



Fig. V-5. Sway vs. Time Y(0) = 0.36, YD = 0.3

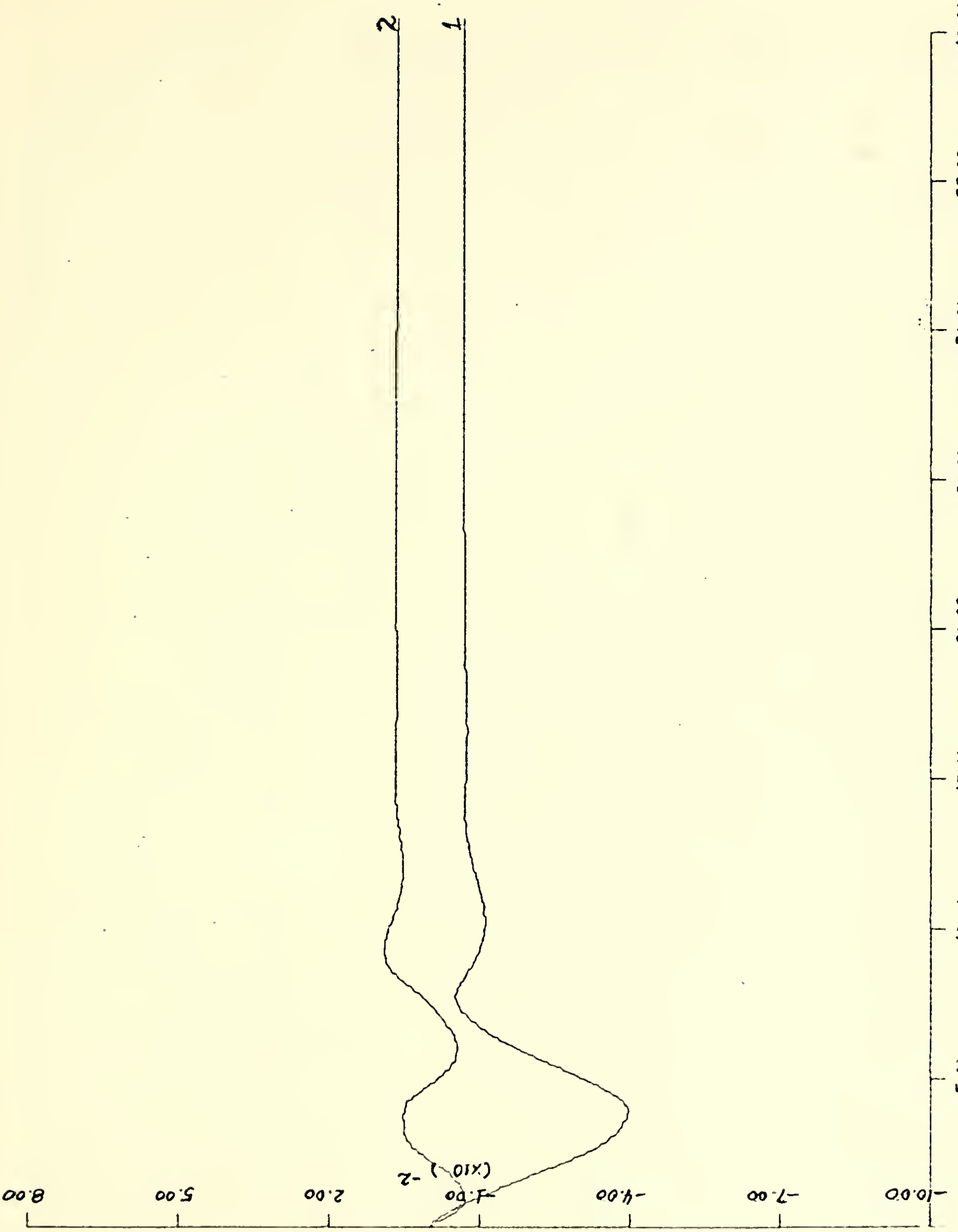



Fig. V-6. Yaw vs. Time Y(0) = 0.36, YD = 0.3

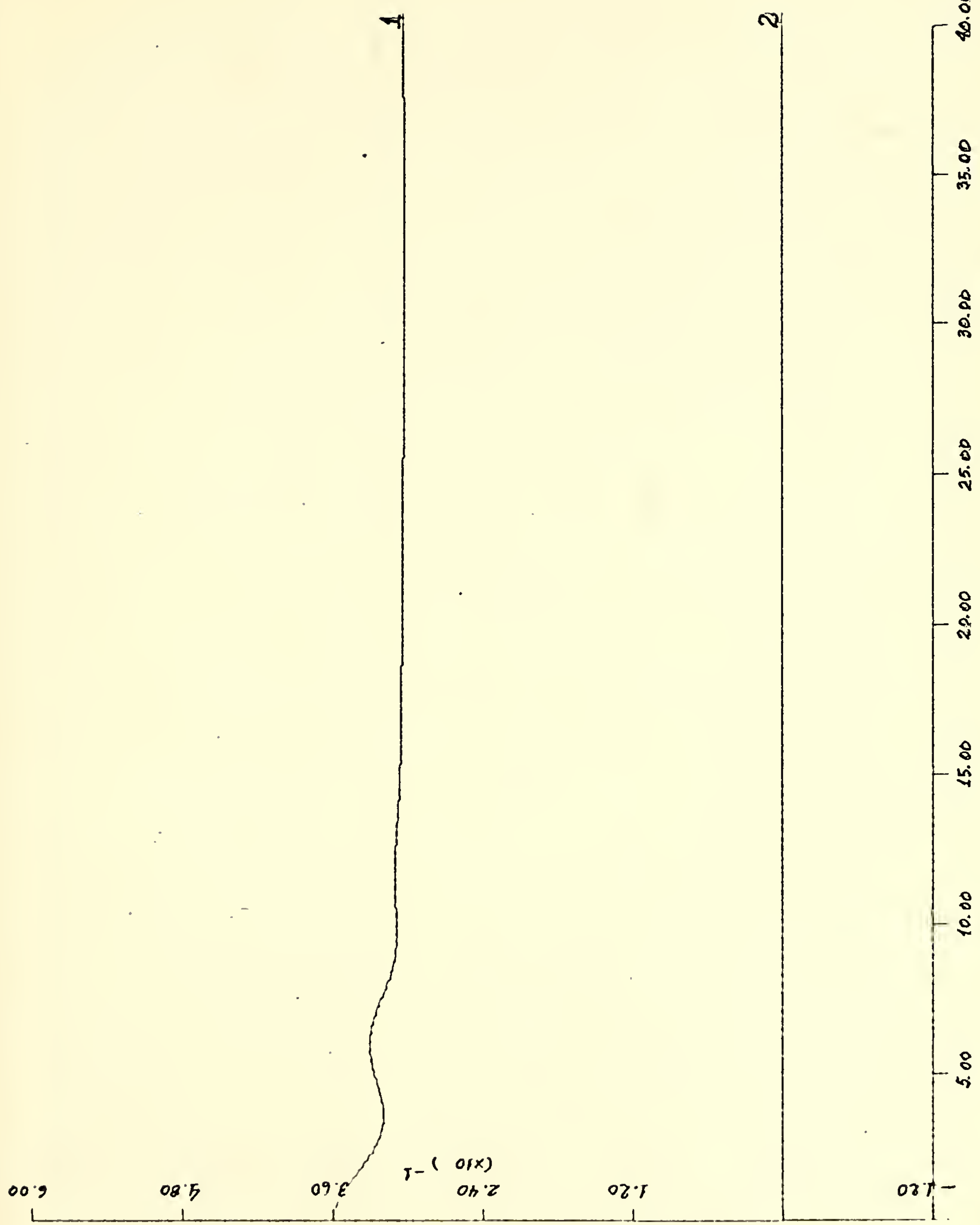



Fig. V-7. Transverse and Longitudinal Separations vs. Time

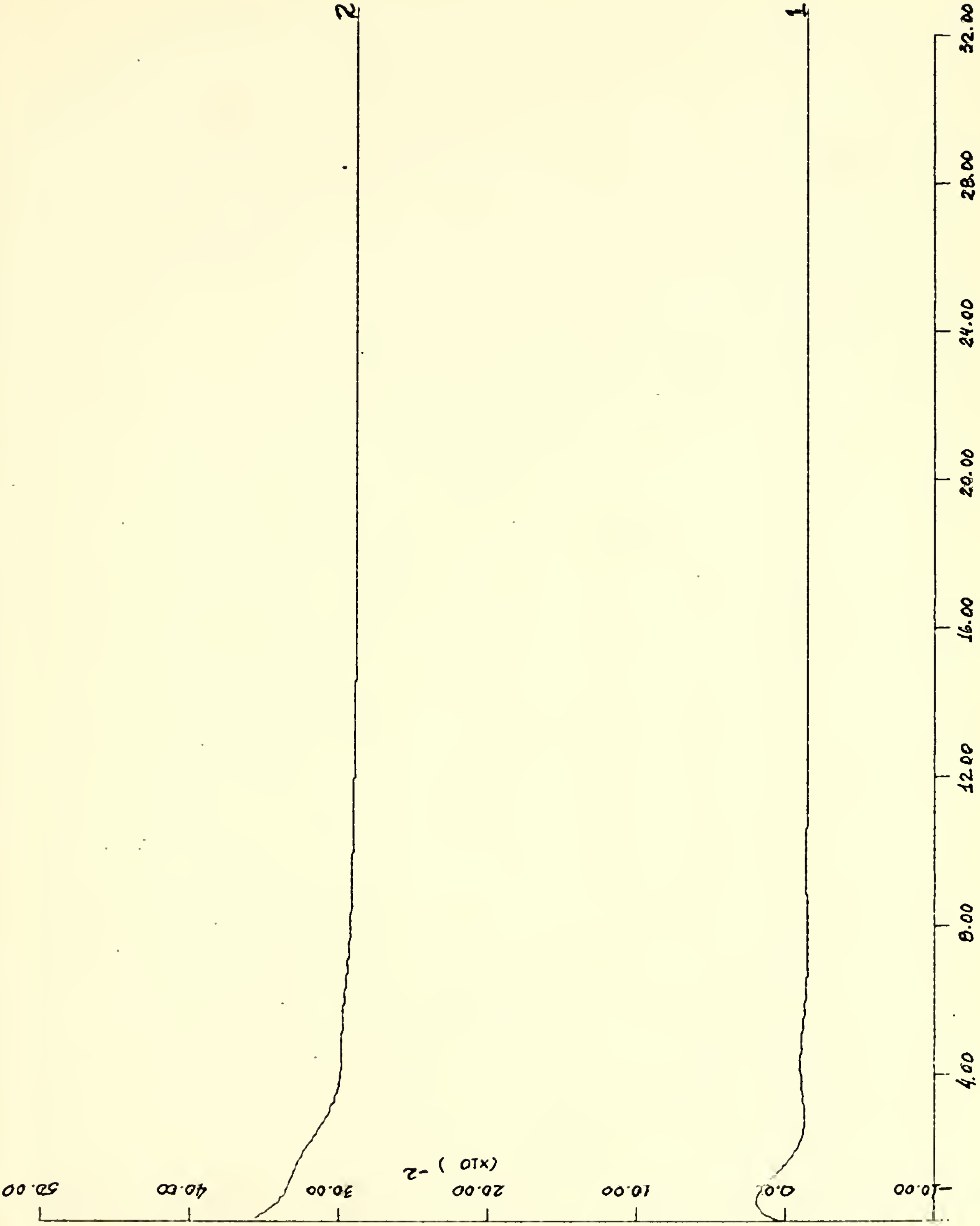



Fig. V-8. Sway vs. Surge Y(0) = 0.36, YD = 0.3

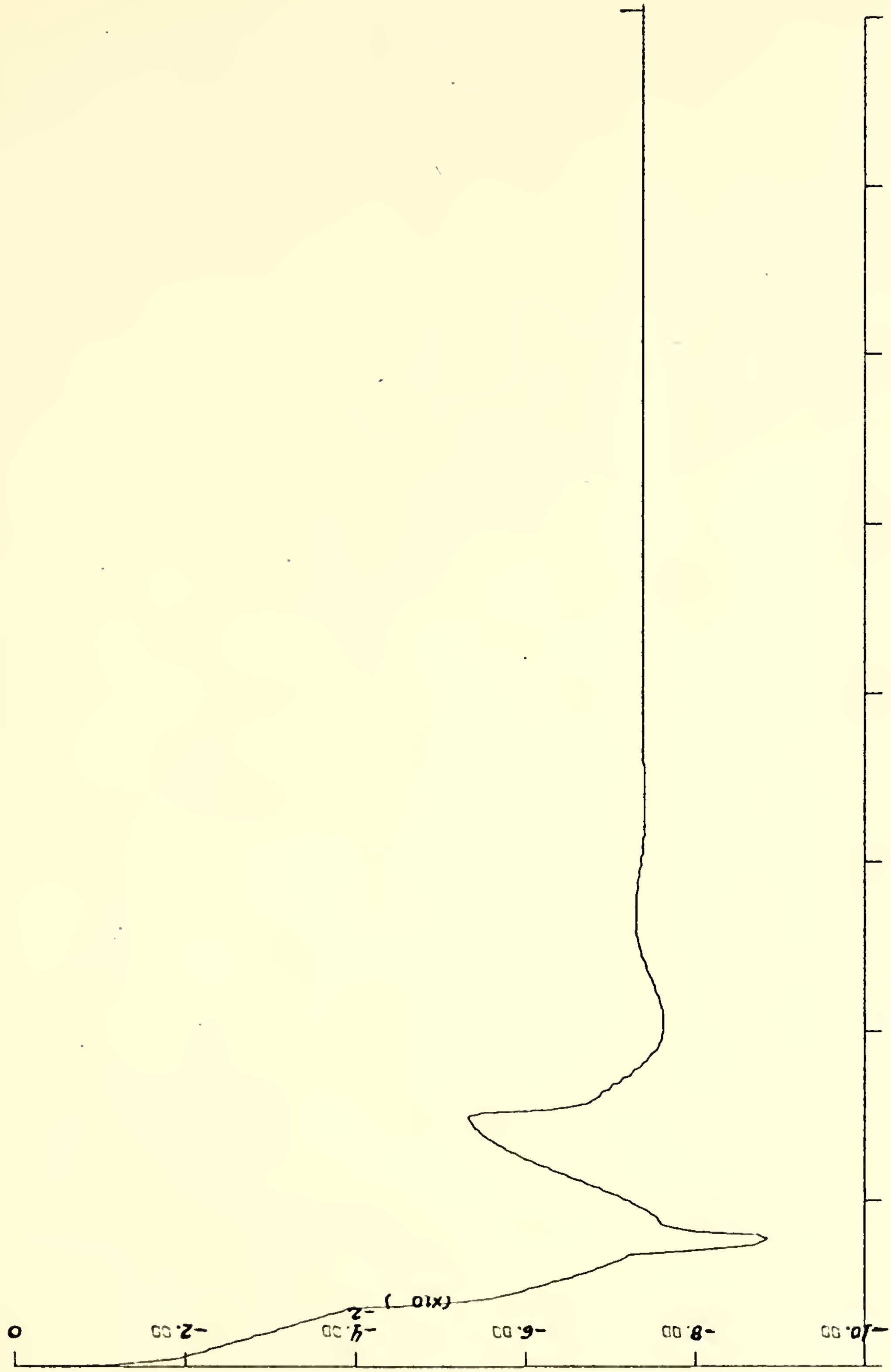



Fig. V-9. Rudder Deflection of the Leading Ship vs. Time

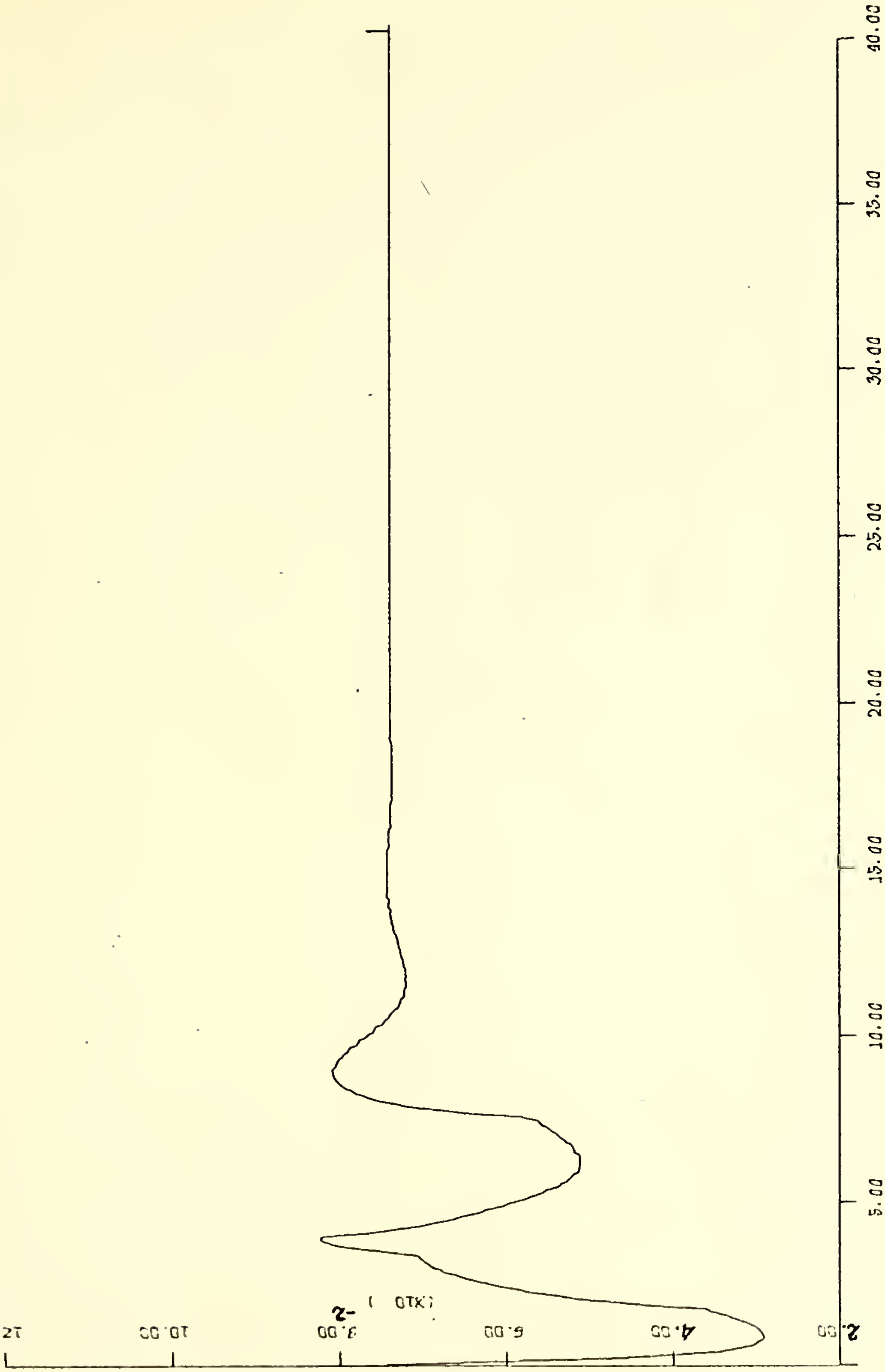



Fig. V-10. Rudder Deflection of the Tracking Ship vs. Time

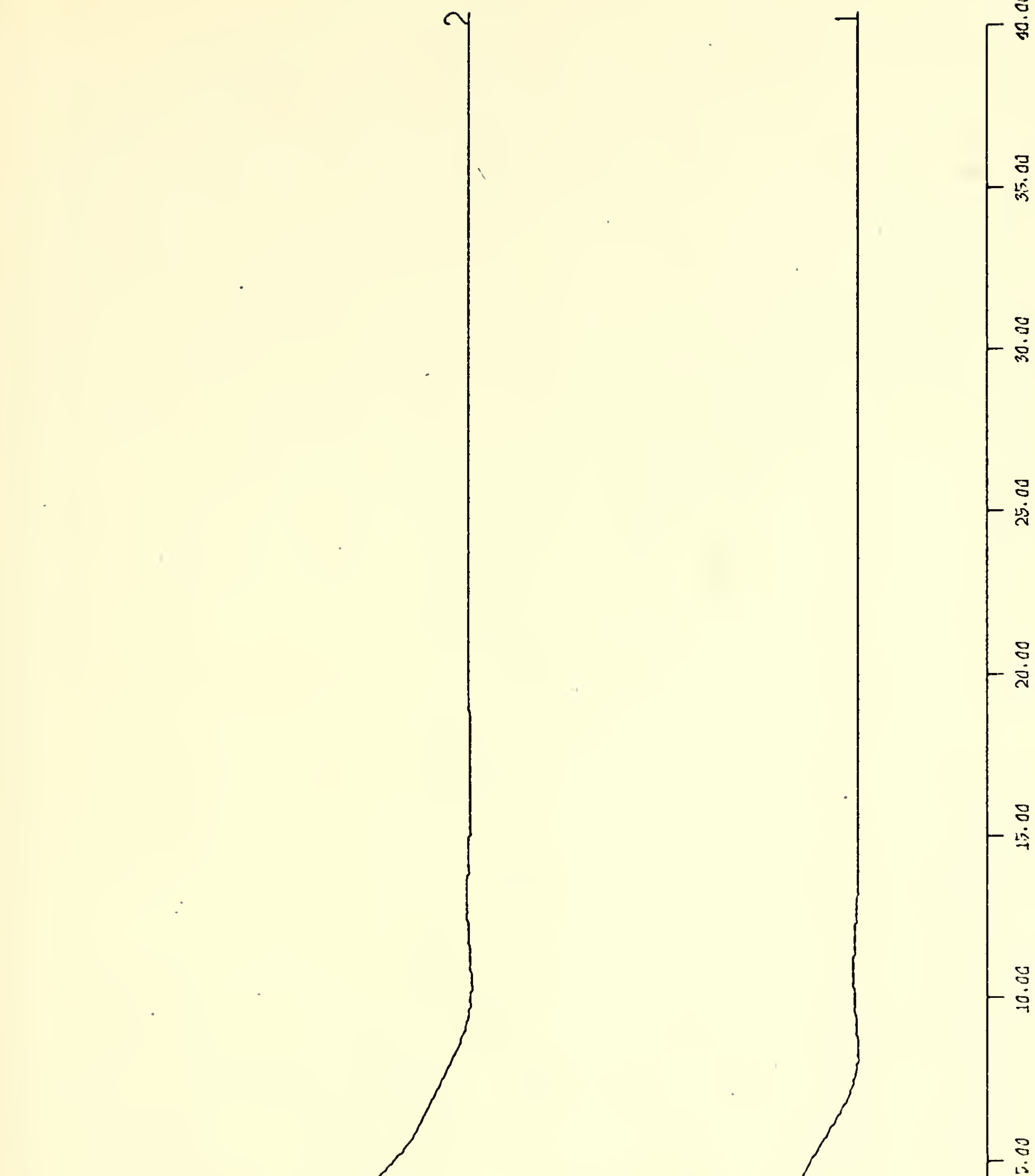



Fig. V-11. Sway vs. Time Y(0) = 0.4, YD = 0.24

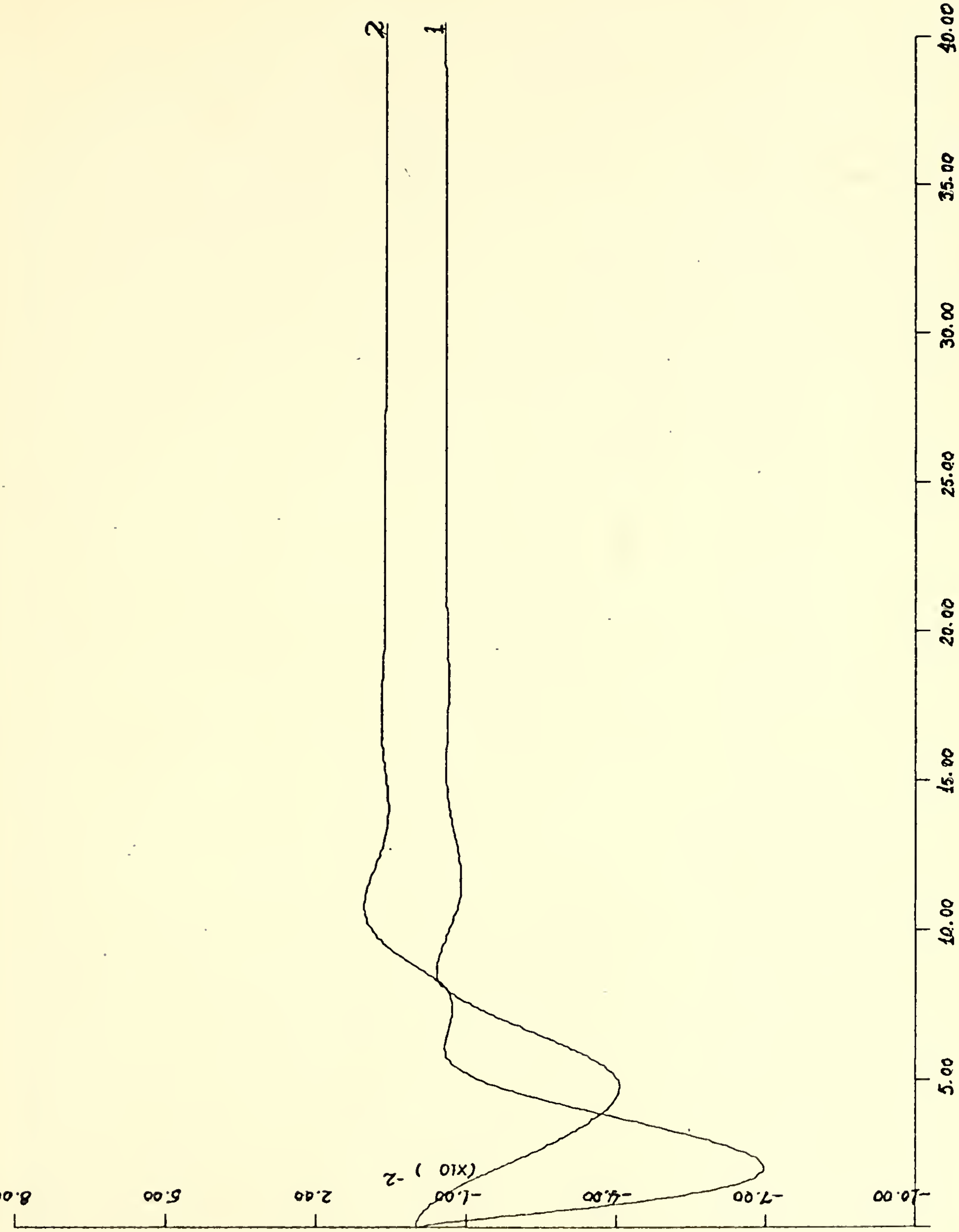



Fig. V-12. Yaw vs. Time Y(0) = 0.4, YD = 0.24

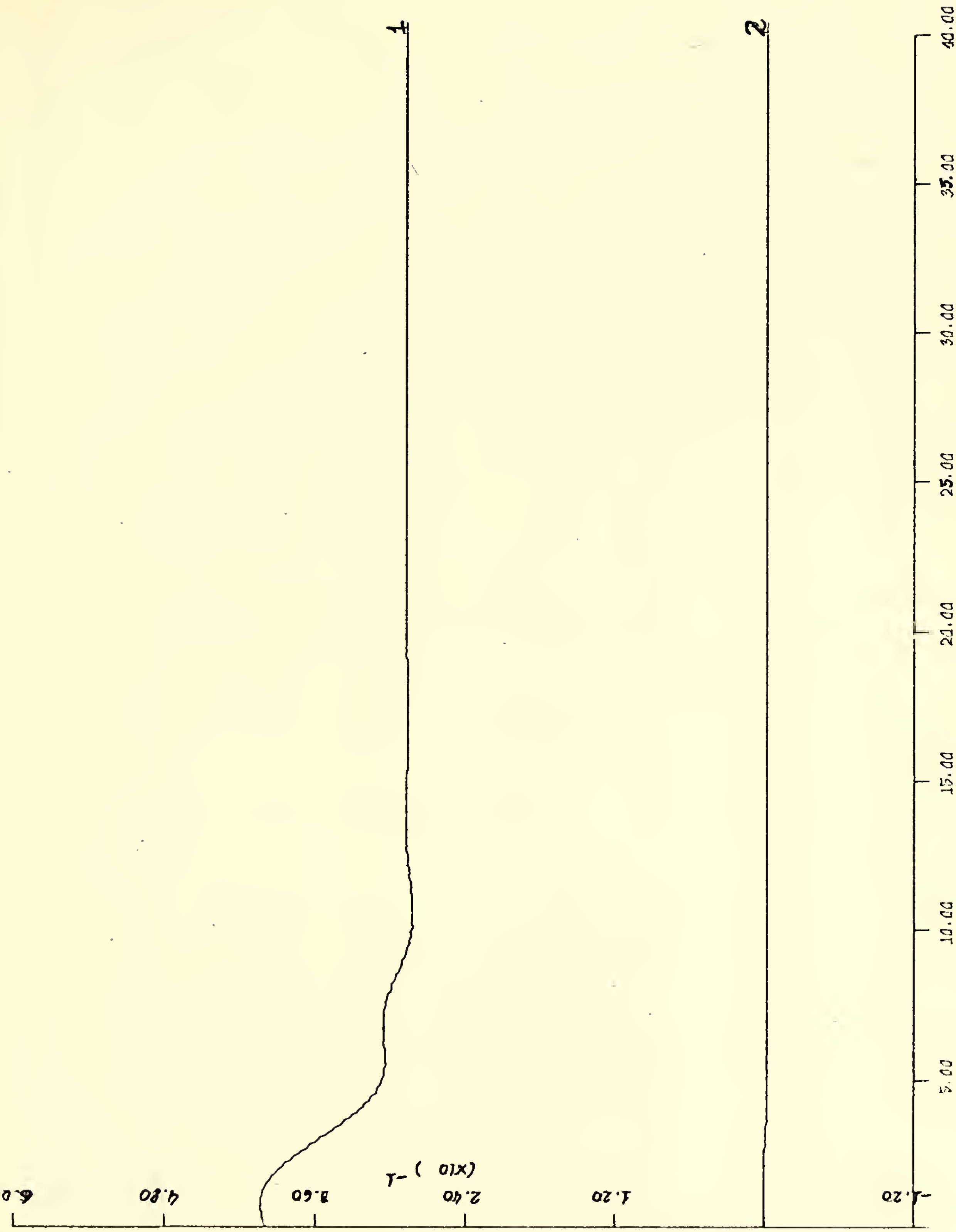



Fig. V-13. Transverse and Longitudinal Separations vs. Time

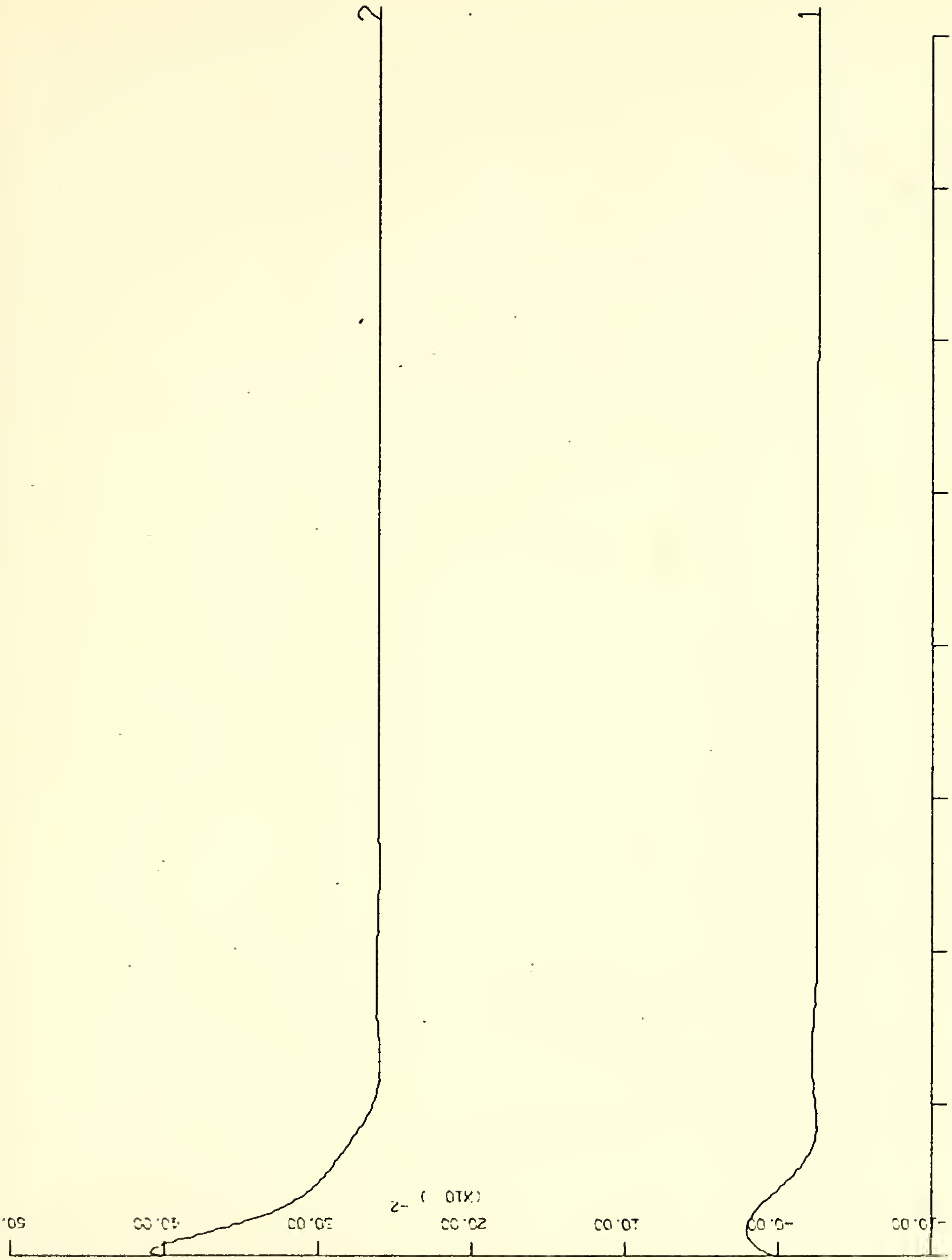



Fig. V-14.  Sway vs. Surge Y(0) = 0.4, YD = 0.24

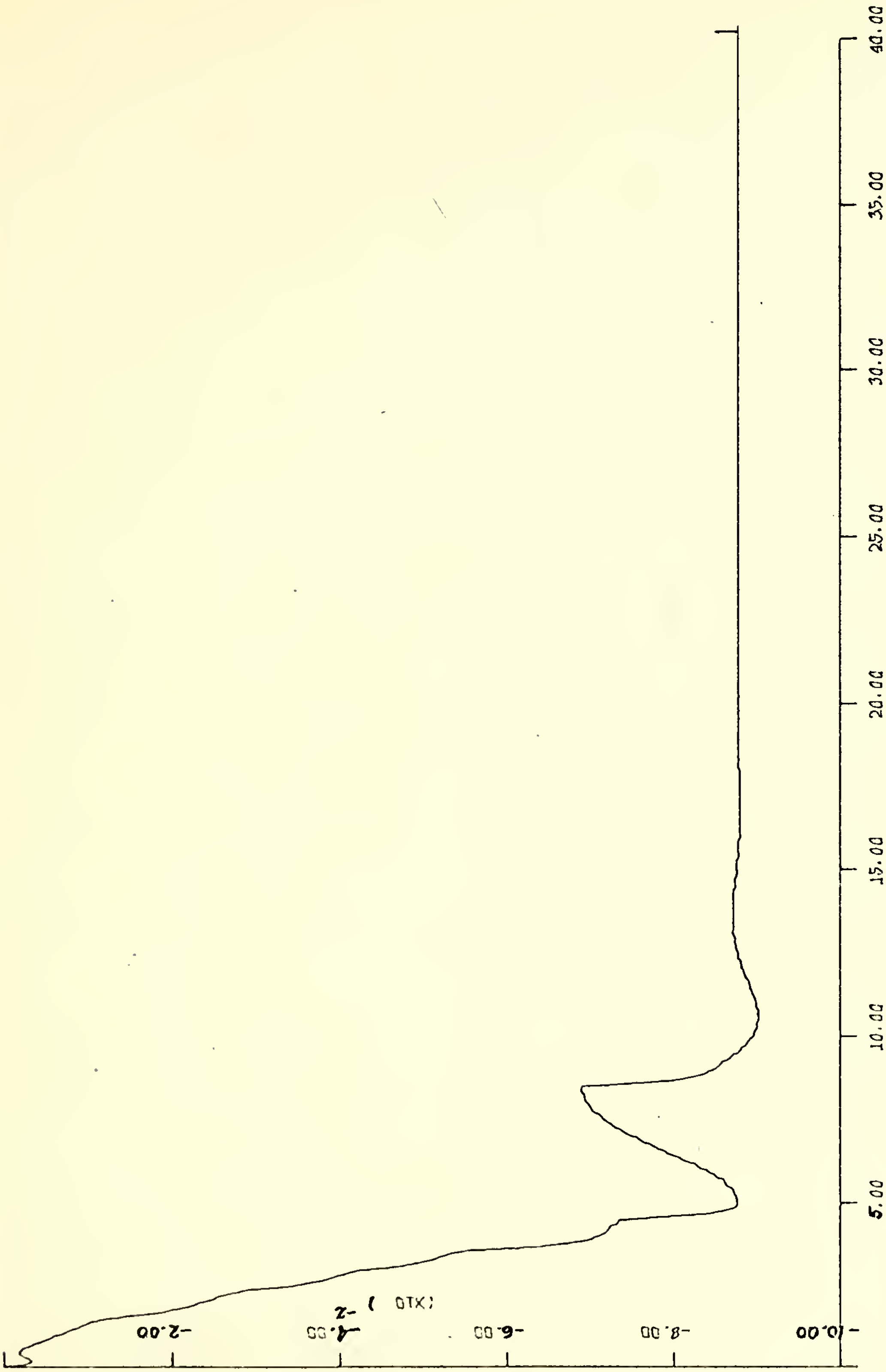



Fig. V-15. Rudder Deflection of the Leading Ship vs. Time

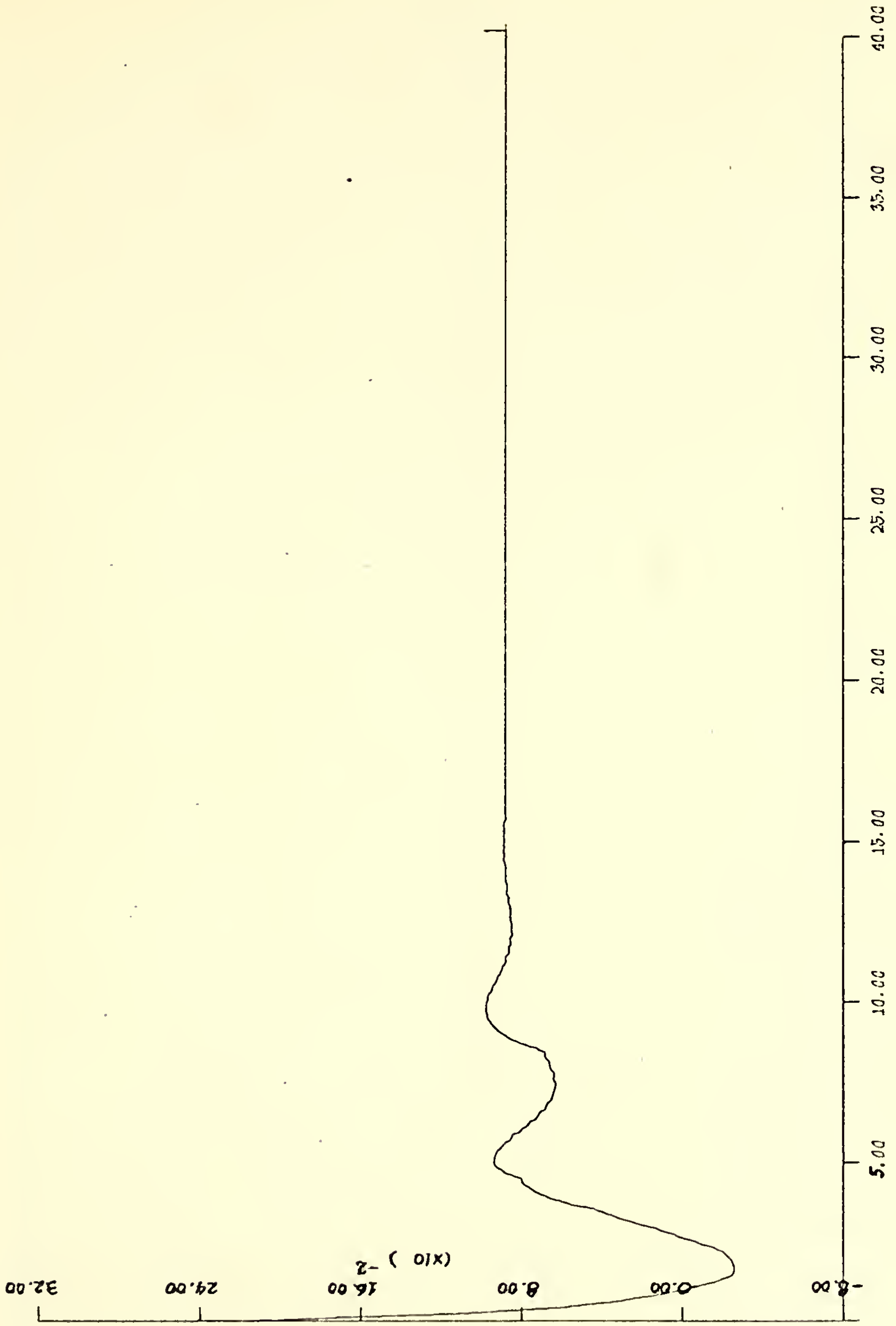



Fig. V-16. Rudder Deflection of the Tracking Ship vs. Time

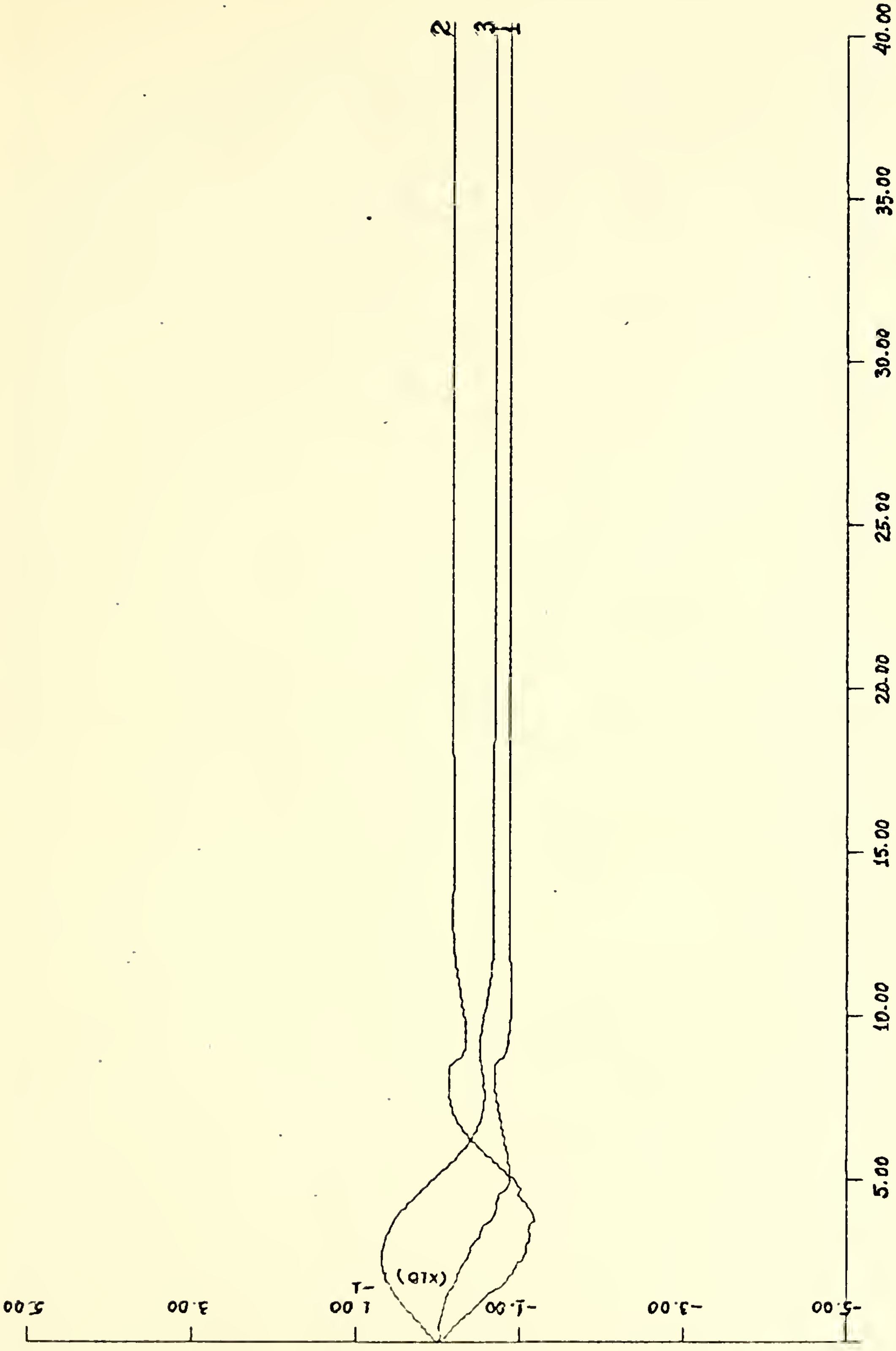



Fig. V-17. Action of the Control Loops on the Leading Ship Rudder Angle

1-Rudder Angle; 2-Course Control; 3-Distance Control

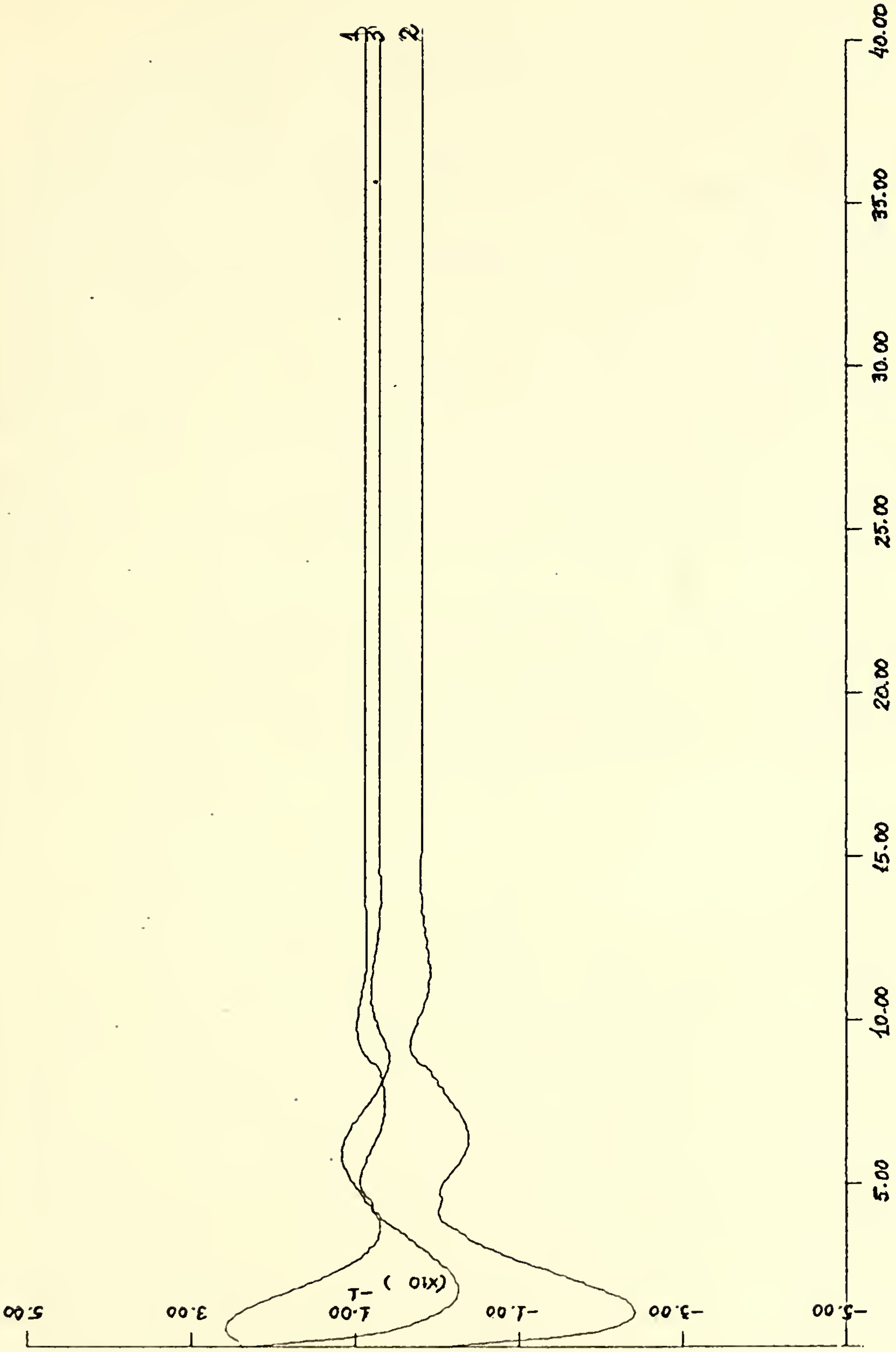



Fig. V-18. Action of the Control Loops on the Tracking Ship Rudder Angle

# VI. CONCLUSIONS

It has been shown that a realistic problem in which the plant dynamics is modelled by a set of non-linear differential equations, coupled by terms that require experimental evaluation and do not have a clear analytical expression, can be approximated by linearization techniques to make possible the use of available linear theory, as done in the steady state decoupling investigation. Integrators were not required in this case for achievement of that performance, but if they were required, the design of the compensator would follow the same steps described in sections III through V.

The parameter optimization method used in this work was shown to be adequate for designing a compensator for plants with complicated mathematical models and, by extension, for problems where the classical methods are not feasible.

## A. FEASIBILITY

The analysis carried out in section V-b leads to the conclusion that the proposed controller has a performance that satisfies the requirements and constraints imposed by the replenishment at sea operation. In particular, crucial problems pertaining to the manual control, such as the exact timing of the command for the rudder swing, and the value of the final rudder deflection (to compensate the interaction forces and moments, so that the ships steam in parallel courses), are correctly solved. The resulting maneuver is safely and efficiently executed.

Implementation requires sensor devices for yaw angles, distances between the ships, and their rates (such as gyroscopes and radars), and a

multiple channel communication link, where the current values of the measured quantities could be exchanged. The rudder command signals would be generated by the conversion of on-line computation results, and transmitted by one of the channels. A frequency multiplexed UHF-FM system appears to be suitable for the case.

B. RECOMMENDATION FOR FURTHER STUDIES

To extend the investigation of the replenishment at sea simulation, some related topics should be carried on in further analysis:

1. The possible simplifications in the control loops, such as making the leading ship insensitive to the rate of change of distance, to the distance itself or to both;

2. The study of underway replenishment operation involving unequal ships;

3. The implementation of the control loops in a hybrid computer and simulation of the controlled plant in real time;

4. The effects of waves, wind and sea states;

5. The design of an automatic controller for thrust so that the approach and departure phases can be analyzed.

## APPENDIX A

## STEADY STATE DECOUPLING OF MULTIVARIABLE SYSTEMS

Consider the system configuration in Figure A-1 where Gp is the m - input - n - output $n_x m$ transfer function matrix of the plant and Gc is the n-input - m-output $m_x n$ transfer function matrix of the compensator to be designed. Complete controllability and observability [8] are assumed. $\underset{\sim}{R}(s)$ and $\underset{\sim}{Y}(s)$ are respectively the n x 1 input and output vectors.

The closed loop transfer function matrix $\underset{\sim}{F}$ can be expressed by

$$\underset{\sim}{F} = \left[ \underset{\sim}{I} + \underset{\sim}{G_p} \underset{\sim}{G_c} \right]^{-1} \underset{\sim}{G_p} \underset{\sim}{G_c} \qquad \text{(A-1)}$$

For total decoupling, $\underset{\sim}{F}$ must be a diagonal matrix; in general, linear state variable feedbacks have been used to accomplish this. The advantages of total decoupling are opposed to some loss of freedom when stability is the point; but if it is required to decouple only the steady states, the classical cascade compensator with unity feedback can be used.

Equation (A-1) can be manipulated to give

$$\underset{\sim}{F} = \underset{\sim}{I} - \left( \underset{\sim}{I} + \underset{\sim}{G_p} \underset{\sim}{G_c} \right)^{-1} \qquad \text{(A-2)}$$

showing that the entries of $\underset{\sim}{F}$ depend in a simple way on the cofactors of the matrix $(\underset{\sim}{I} + \underset{\sim}{Gp}\ \underset{\sim}{Gc})$; this will be a useful result for the next steps.

A system like that of Figure I-1 is defined[3] to be steady-state decoupled if and only if it is a symptotically stable and

---

[3]The definitions and derivations mentioned in this Appendix were all obtained in Reference 7 and included in this work to justify the behavior adopted in Section III-C as well as to serve as a short reference in the subject matter.

$$\lim_{s\to 0} s \sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ij}(s) Y_j(s) = 0 \qquad \text{(A-3)}$$

Where

$f_{ij}$ is the $ij^{th}$ entry of $\underset{\sim}{F}(s)$

$Y_{ij} = s^{kj-1}$ is the $j^{th}$ input

Equation (A-3) can be expressed in terms of the cofactors of $\underset{\sim}{F}$ by

$$\lim_{s\to 0} \frac{1}{s^{kj-1}} \cdot \frac{\operatorname{Cof}\left[\underset{\sim}{I} + \underset{\sim}{G}p\,\underset{\sim}{G}c\right]_{ji}}{\operatorname{Det}\left[\underset{\sim}{I} + \underset{\sim}{G}p\,\underset{\sim}{G}c\right]} = 0 \qquad \text{(A-4)}$$

The plant type number matrix $\underset{\sim}{T}_p$ is obtained by separating in each entry of $\underset{\sim}{G}_p$ the powers of s from the rest part of the transfer functions,

$$g_{pij} = s^{-t_{pij}} \cdot g'_{pij}$$

What gives the $ij^{th}$ entry of $\underset{\sim}{T}_p$.

Similarly the compensator type number matrix $\underset{\sim}{Tc}$ will be obtained by

$$g_{cij} = s^{-t_{cij}} \cdot g'_{cij}$$

For a 2 x 2 plant and a diagonal 2 x 2 compensator,

$$\underset{\sim}{T}_p = \begin{bmatrix} t_{p11} & t_{p12} \\ t_{p21} & t_{p22} \end{bmatrix}, \quad \underset{\sim}{T}_c = \begin{bmatrix} t_{c11} & 0 \\ 0 & t_{c22} \end{bmatrix} \qquad \text{(A-6)}$$

Manipulating (A-4) with (A-5), and letting Nij to be the highest factorable power of s in the numerator corresponding to the $ij^{th}$ cofactor and the M to be highest factorable power of s in the denominator of (A-4), for all i,j, i≠j, it can be shown that

$$N_{12} = \operatorname{Max}\,(t_{p21} + t_{c11}) = t_{p21} + t_{c11}$$

$$N_{21} = \operatorname{Max}\,(t_{p12} + t_{c22}) = t_{p12} + t_{c22}$$

$$M = \operatorname{Max}\{(t_{p11} + t_{c11}), (t_{p22} + t_{c22}), (t_{p11} + t_{p22} + t_{c11} + t_{c22}), (t_{p12} + t_{p21} + t_{c11} + t_{c22})\} \qquad \text{(A-7)}$$

and that for steady state decoupling any of the following four sets of criteria can be used:

a) $M > \text{Max}\ (N_{12}, N_{21})$

b) $M < 0,\ M > N_{12},\ N_{21} < 0$

c) $M < 0,\ M > N_{21},\ N_{12} < 0$

d) $M < 0,\ N_{12} < 0,\ N_{21} < 0$

The best choice among those four sets will depend on Tp; since $tp_{ij}$ are known from the plant, the only unknowns are $tc_{ij}$, which must be chosen so that the solution is physically possible (not introducing pure differentiators) and as simple as possible: tcij will be the minimum positive integer satisfying the criteria stated above.

For a 3 x 3 plant with a 3 x 3 diagonal compensator, M and Nij can be obtained as mentioned for the previous case, becoming:

$$M = \text{Max}\left\{(t_{p11}+t_{c11}), (t_{p22}+t_{c22}), (t_{p33}+t_{c33}), (t_{p11}+t_{p22}+t_{c11}+t_{c22}+\ldots)\right\}$$

$$N_{13} = \text{Max}\left\{(t_{p31}+t_{c11}), (t_{p31}+t_{p22}+t_{c11}+t_{c22}), (t_{p32}+t_{p21}+t_{c11}+t_{c22})\right\}$$

$$N_{21} = \text{Max}\left\{(t_{p12}+t_{c22}), (t_{p12}+t_{p33}+t_{c22}+t_{c33}), (t_{p13}+t_{p32}+t_{c22}+t_{c33})\right\}$$

$$N_{23} = \text{Max}\left\{(t_{p32}+t_{c22}), (t_{p32}+t_{p11}+t_{c11}+t_{22}), (t_{p31}+t_{p12}+t_{c11}+t_{c22})\right\}$$

$$N_{31} = \text{Max}\left\{(t_{p13}+t_{c33}), (t_{p13}+t_{p22}+t_{c22}+t_{c33}), (t_{p23}+t_{p12}+t_{22}+t_{c33})\right\}$$

$$N_{32} = \text{Max}\left\{(t_{p23}+t_{c33}), (t_{p23}+t_{p11}+t_{c11}+t_{c33}), (t_{p21}+t_{p13}+t_{c11}+t_{c33})\right\} \tag{A-9}$$

In this case 64 sets of criteria can be established. The corresponding to (A-8-a) is

$M > \text{Max}\ (Nij,\ i,\ j = 1,2,3,\ i \neq j)$

A general expression for a n x n plant with a diagonal n x n compensator, and further studies of stability and design can be found in the reference.

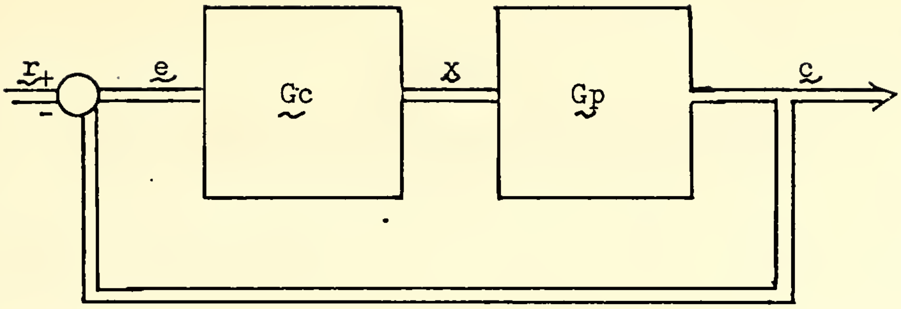



Fig. A-1. Unity Feedback Multivariable System With Cascade Compensator

APPENDIX B

THE ASSIGNED RESPONSE FOR THE RECEIVING SHIP

Equation (II-17) gives

$$\dot{y} = u \sin \psi + V \cos \psi \qquad \text{(B-1)}$$

For small values of one can take

$$\sin \psi \simeq \psi$$

$$\cos \psi \simeq 1$$

so that (B-1) becomes, with U = 1

$$\dot{y} \simeq \psi + V \qquad \text{(B-2)}$$

Replacing $\psi$ and V by the transfer functions (II-22),

$$\dot{y} = \frac{K_r(s+z_r) + s\,K_v\,(s+z_v)}{s(s^2+ps+q)}\,\delta(s) \qquad \text{(B-3-a)}$$

and

$$y = \frac{\dot{y}}{s} = \frac{K_r(s+z_r) + s\,K_v\,(s+z_v)}{s^2(s^2+ps+q)}\,\delta(s) \qquad \text{(B-3-b)}$$

The block diagram for a ship with distance keeping loop is shown in Figure B-1, from which

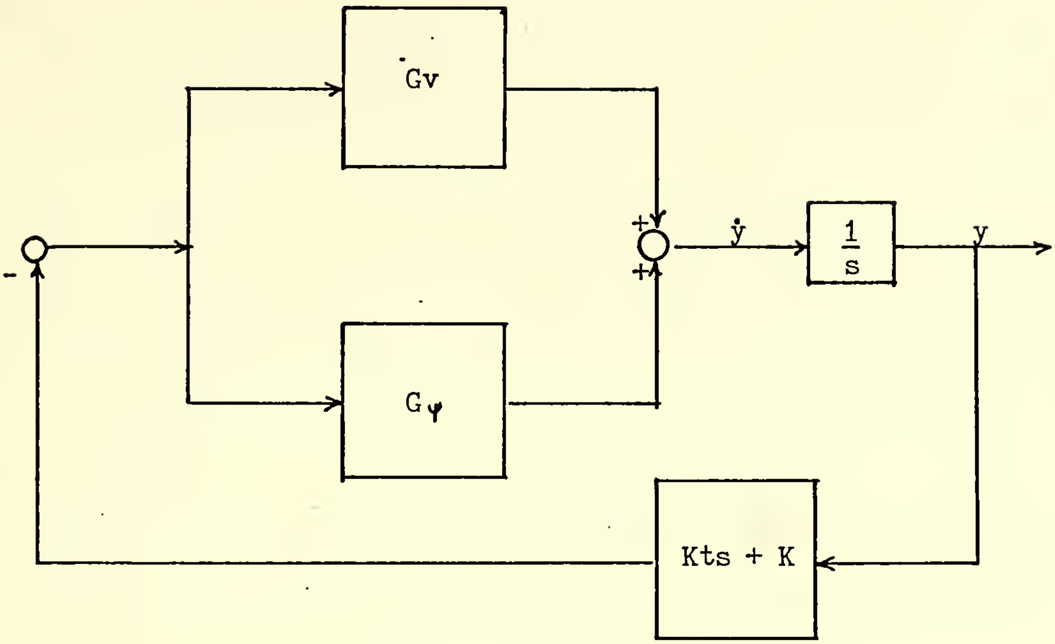
$$\delta(s) = (K_{ty}\,s + K_y)\,y \qquad \text{(B-4)}$$

and the closed loop transfer function is then

$$F(s) = \frac{G(s)}{1 - G(s)\cdot H(s)}$$

Using equations B-3-a and b comes

$$F(s) = \frac{K_v s^2 + (K_v z_v + K_r)s + K_r z_r}{s^4 + (p - K_v K_{ty})s^3 + [q - K_v K_y - K_{ty}(K_v z_v + K_r)]s^2 + [-K_y(K_v z_v + K_r) - K_{ty} K_r z_r]s - K_r K_y z_r} \qquad \text{(B-5)}$$



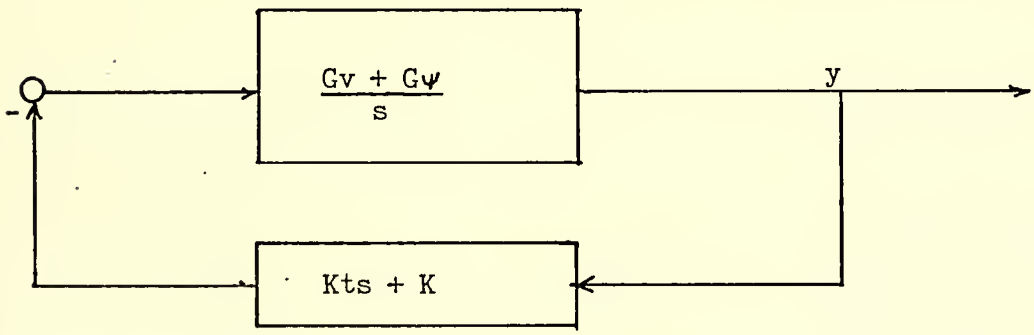




Fig. B-1. A) Distance Keeping Loop
B) Equivalent Block Diagram

and the characteristic equation is

$$s^4 + (p - K_v K_{ty}) s^3 + [q - K_v K_y - K_{ty}(K_v z_v + K_r)] s^2 + \\ + [-K_y(K_v z_v + K_r) - K_{ty} K_r z_r] s - K_r z_r K_y = 0 \qquad \text{(B-6)}$$

Replacing values of Table II-5,

$$s^4 + (0.68467 - 0.21447 K_{ty}) s^3 + [1.016959 - 0.21447 K_y + \\ + 0.67774 K_{ty}] s^2 + [2.4775 K_{ty} + 0.67774 K_y] s + 2.4775 K_y = 0$$

The pairs of values of $K_y$ and $K_{ty}$ which yield a critically damped system are readily found using parameter plane techniques [16]. A sample of such values is shown in Table B-1. Taking the pair

$K_y = 0.0025074$

$K_{ty} = 0.063325$

the ideal response was simulated using a DSL-360 computer program CP-V which is simply a modification of program I. In this case equations (II-12) become

$$IF_1 = Y_\delta [K_y (y - y_d) + K_{ty} \dot{y}]$$

$$IF_2 = N_\delta [K_y (y - y_d) + K_{ty} \dot{y}]$$

Figure B-2 show the desired trajectory for station changing from an initial position

$x(o) = 0, \ y(o) = 0.2$

to the final position

$x(t_f) = 0, \ y(t_f) = 0.1$

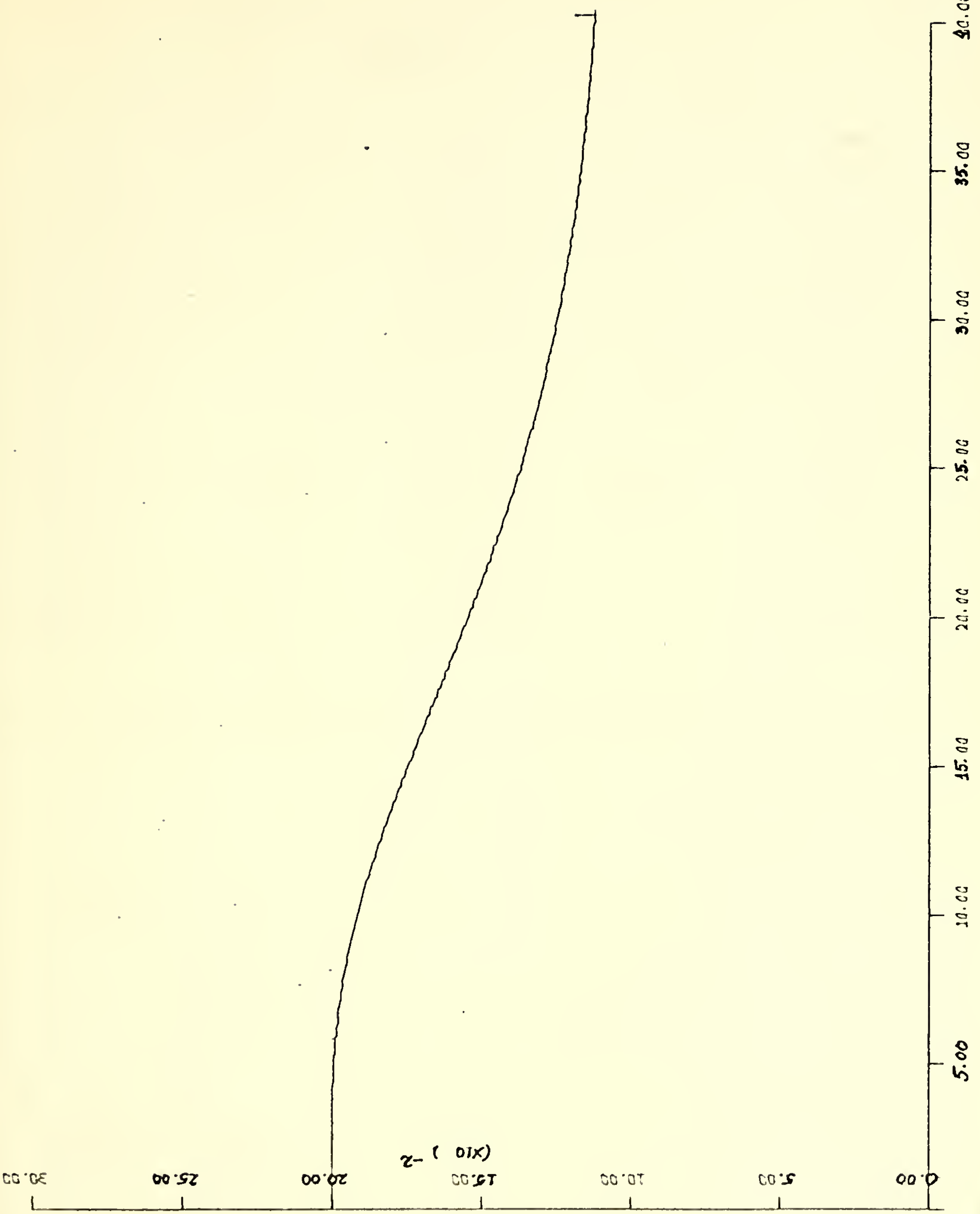



Fig. B-2. Tracking Ship's Desired Trajectory

TABLE B-1

VALUES OF $K_y$ and $K_{ty}$ for = 1.0

(SAMPLE)

| $K_y$ | $K_{ty}$ | n |
|---|---|---|
| 0.40717E-04 | 0.81590E-02 | 0.10000E-01 |
| 0.42025E-04 | 0.82888E-02 | 0.10160E-01 |
| 0.43375E-04 | 0.84206E-02 | 0.10323E-01 |
| 0.44769E-04 | 0.85546E-02 | 0.10488E-01 |
| 0.46207E-04 | 0.86906E-02 | 0.10656E-01 |
| 0.47692E-04 | 0.88288E-02 | 0.10826E-01 |
| 0.49224E-04 | 0.89692E-02 | 0.10999E-01 |
| 0.50805E-04 | 0.91118E-02 | 0.11175E-01 |
| 0.52437E-04 | 0.92567E-02 | 0.11354E-01 |
| 0.54121E-04 | 0.94038E-02 | 0.11536E-01 |

```
//LIMA$1 JOB (0709,0500,EA24),'LIMA',TIME=4,MSGLEVEL=(0,0)
// EXEC DSL
//DSL.INPUT DD *



*       COMPUTER PROGRAM I


*       LINEAR RESPONSE OF THE MARINER

INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=1

*       HYDRODYNAMIC COEFFICIENTS

CONST NR=-0.00227,NV=-0.00351,NVD=-0.000197
CONST MYVD=0.015,MYR=0.0051,IZNRD=0.00068,MXUD=0.0085
CONST YV=-0.01243,XU=-0.0012,YRD=-0.00027
CONST YDELR=-0.0027,NDELR=-0.00126,XDELR=0.0

*       APPLIED RUDDER DEFLECTION

PARAM D1=0.1

*       INITIAL CONDITIONS

INCON X0=0.,Y0=0.

INITIAL

*       CALCULATION OF THE COEFFICIENTS

        A11=MYVD
        B11=-YV
        A21=-YRD
        B21=MYR
        A12=-NVD
        B12=-NV
        A22=IZNRD
        B22=-NR
        A33=MXUD
        B33=-XU
        NC=-XU
        KA1=-YDELR
        KB1=NDELR
        KC1=XDELR
        C=A11*A22-A12*A21
        IF1=KA1*D1
        IF2=KB1*D1
        IF3=KC1*D1+NC
```

```
DERIVATIVE

*       TIME DOMAIN SIMULATION

        I1=-B11*ADOT-B21*BDOT+IF1
        I2=-B12*ADOT-B22*BDOT+IF2
        I3=-B33*CDOT+IF3
        ADDOT=(I1*A22-I2*A21)/D
        BDDOT=(I2*A11-I1*A12)/D
        CDDOT=I3/A33
        ADOT=INTGRL(0.,ADDOT)
        BDOT=INTGRL(0.,BDDOT)
        CDOT=INTGRL(0.,CDDOT)
        A=INTGRL(0.,ADOT)
        B=INTGRL(0.,BDOT)
        C=INTGRL(0.,CDOT)
        XDOT=CDOT*COS(B)-ADOT*SIN(B)
        YDOT=CDOT*SIN(B)+ADOT*COS(B)
        X=INTGRL(X0,XDOT)
        Y=INTGRL(Y0,YDOT)
        YAW=B
        SWAY=Y
        SURGE=X
SAMPLE
CONTRL  FINTIM=30.,DELT=0.04,DELS=0.04
PREPAR  1.,SURGE,SWAY,YAW
GRAPH TIME,YAW
GRAPH TIME,SWAY
GRAPH  SAME,10,10,SURGE,SWAY
PRPLOT  ONLY
        CALL  DRWG(1,1,TIME,YAW)
        CALL  DRWG(2,1,SURGE,SWAY)
TERMINAL
        CALL  ENDRW(NPLOT)
END
STOP
//PLOT.SYSIN DD *
```

```
//LIMA$2 JOB (0709,0500,EA24),'LIMA',TIME=4,MSGLEVEL=(0,0)
// EXEC DSL
//DSL.INPUT DD *



*       COMPUTER PROGRAM II

*       UNCOMPENSATED SYSTEM RESPONSE

*       HYDRODYNAMIC COEFFICIENTS

PARAM MXUD=-0.0085,XU=-0.0012
PARAM MYR=-0.0051,YRD=-0.00027
PARAM YV=-0.01243,MYVD=-0.015
PARAM NVD=-0.000197,NV=-0.00351
PARAM NR=-0.00227,IZNRD=-0.00068
PARAM YDEL=0.0027,NDEL=-0.00126

*       INITIAL SEPARATION BETWEEN THE SHIPS

INCON X10=0.,X20=0.
INCON Y10=0.,Y20=0.2

PARAM YI=0.,NI=0.
PARAM U1=1.,U2=1.
PARAM DD1=0.0,DD2=0.0
INITIAL

*       CALCULATION OF THE COEFFICIENTS

        A11=-MYVD
        B11=-YV
        C11=0.
        A21=-YRD
        B21=-MYR
        C21=0.
        A12=-NVD
        B12=-NV
        C12=0.
        A22=-IZNRD
        B22=-NR
        C22=0.
        A33=-MXUD
        B33=-XU
        NC=-XU
        KA=YDEL
        KB=NDEL
        D=A11*A22-A12*A21

*       INITIAL LATERAL SEPARATION

        DY0=Y20-Y10
        DX0=X20-X10

        CALL FORCES(DX0,DY0,YI,NI)
```

```
DERIVATIVE

*       SIMULATION

        YDOT1=CDOT1*SIN(B1)+ADOT1*COS(B1)
        YDOT2=CDOT2*SIN(B2)+ADOT2*COS(B2)
        XDOT1=CDOT1*COS(B1)-ADOT1*SIN(B1)
        XDOT2=CDOT2*COS(B2)-ADOT2*SIN(B2)
        ADD1=(A22*I11-A21*I21)/D
        ADD2=(A22*I12-A21*I22)/D
        BDD1=(A11*I21-A12*I11)/D
        BDD2=(A11*I22-A12*I12)/D
        CDD1=I31/A33
        CDD2=I32/A33
        ADOT1=INTGRL(0.,ADD1)
        ADOT2=INTGRL(0.,ADD2)
        BDOT1=INTGRL(0.,BDD1)
        BDOT2=INTGRL(0.,BDD2)
        CDOT1=INTGRL(0.,CDD1)
        CDOT2=INTGRL(0.,CDD2)
        A1=INTGRL(0.,ADOT1)
        A2=INTGRL(0.,ADOT2)
        B1=INTGRL(0.,BDOT1)
        B2=INTGRL(0.,BDOT2)
        C1=INTGRL(0.,CDOT1)
        C2=INTGRL(0.,CDOT2)
        Y1=INTGRL(Y10,YDOT1)
        Y2=INTGRL(Y20,YDOT2)
        X1=INTGRL(X10,XDOT1)
        X2=INTGRL(X20,XDOT2)
        I11=-B11*ADOT1-C11*A1-B21*BDOT1-C21*B1+IF11
        I12=-B11*ADOT2-C11*A2-B21*BDOT2-C21*B2+IF12
        I21=-B12*ADOT1-C12*A1-B22*BDOT1-C22*B1+IF21
        I22=-B12*ADOT2-C12*A2-B22*BDOT2-C22*B2+IF22
        I31=-B33*CDOT1+IF31
        I32=-B33*CDOT2+IF32
        AF11=REALPL(0.,0.1,KA*DD1)
        AF12=REALPL(0.,0.1,KA*DD2)
        AF21=REALPL(0.,0.1,KB*DD1)
        AF22=REALPL(0.,0.1,KB*DD2)
        IF11=AF11+YI
        IF12=AF12-YI
        IF21=AF21+NI
        IF22=AF22-NI
        IF31=NC
        IF32=NC

DYNAMIC

*       ACTUAL SEPARATION

        DX=X2-X1
        DY=Y2-Y1

        CALL FORCES(DX,DY,YI,NI)
SAMPLE
PRINT 0.2,DX,DY,YI,NI
PREPAR 0.1,B1,B2,Y1,Y2,DY,DX
CONTRL FINTIM=10.,DELT=0.02,DELS=0.1
GRAPH TIME,Y1,Y2
GRAPH TIME,B1,B2
GRAPH TIME,DY,DX
PRPLCT ONLY
        IF(DY.LE.0.05)WRITE(6,3)
    3   FORMAT(' ','LATERAL SEPARATION LESS THAN 25 FT')
```

```
        CALL  DRWG(1,1,TIME,Y1)
        CALL  DRWG(1,2,TIME,Y2)
        CALL  DRWG(2,1,TIME,B1)
        CALL  DRWG(2,2,TIME,B2)
TERMINAL
        CALL  ENDRW (NPLOT)
END
PARAM Y10=0.,Y20=0.1
END
STOP
//PLCT.SYSIN DD *
```

```
SUBROUTINE FORCES(DX,DY,YD,YND)


TABLE LOOK-UP AND INTERPOLATION

THIS SUBROUTINE GIVES THE VALUES OF THE INTERACTION
FORCES AND MOMENTS BEING EXERTED ON THE LEADING SHIP
AS FUNCTIONS OF THE LONGITUDINAL AND TRANSVERSE
SEPARATIONS BETWEEN THE TWO SHIPS

DIMENSION Z(5,16),W(5,16),X(5),Y(16)
X(1)=-0.2
X(2)=-0.1
X(3)=0.
X(4)=0.1
X(5)=0.2
Y(1)=0.10
Y(2)=0.12
Y(3)=0.14
Y(4)=0.16
Y(5)=0.18
Y(6)=0.2
Y(7)=0.22
Y(8)=0.24
Y(9)=0.26
Y(10)=0.28
Y(11)=0.30
Y(12)=0.32
Y(13)=0.34
Y(14)=0.36
Y(15)=0.38
Y(16)=0.40
Z(1,1)=52.
Z(1,2)=47.
Z(1,3)=43.
Z(1,4)=39.
Z(1,5)=36.
Z(1,6)=34.
Z(1,7)=32.
Z(1,8)=30.
Z(1,9)=28.
Z(1,10)=26.
Z(1,11)=24.
Z(1,12)=22.
Z(1,13)=20.
Z(1,14)=18.
Z(1,15)=16.
Z(1,16)=14.
Z(2,1)=72.
Z(2,2)=64.
Z(2,3)=58.
Z(2,4)=52.
Z(2,5)=46.
Z(2,6)=43.
Z(2,7)=40.
Z(2,8)=37.
Z(2,9)=34.
Z(2,10)=31.
Z(2,11)=28.
Z(2,12)=25.
Z(2,13)=22.
Z(2,14)=19.
Z(2,15)=17.
Z(2,16)=15.
Z(3,1)=86.
```

```
Z(3,2)=75.
Z(3,3)=67.
Z(3,4)=60.
Z(3,5)=53.
Z(3,6)=48.
Z(3,7)=43.
Z(3,8)=39.
Z(3,9)=35.
Z(3,10)=32.
Z(3,11)=29.
Z(3,12)=26.
Z(3,13)=23.
Z(3,14)=20.
Z(3,15)=18.
Z(3,16)=16.
Z(4,1)=89.
Z(4,2)=78.
Z(4,3)=69.
Z(4,4)=60.
Z(4,5)=53.
Z(4,6)=48.
Z(4,7)=43.
Z(4,8)=38.
Z(4,9)=34.
Z(4,10)=31.
Z(4,11)=28.
Z(4,12)=25.
Z(4,13)=22.
Z(4,14)=19.
Z(4,15)=17.
Z(4,16)=15.
Z(5,1)=80.
Z(5,2)=70.
Z(5,3)=63.
Z(5,4)=55.
Z(5,5)=50.
Z(5,6)=45.
Z(5,7)=40.
Z(5,8)=36.
Z(5,9)=33.
Z(5,10)=30.
Z(5,11)=29.
Z(5,12)=26.
Z(5,13)=23.
Z(5,14)=20.
Z(5,15)=18.
Z(5,16)=16.
W(1,1)=-44.
W(1,2)=-38.
W(1,3)=-33.
W(1,4)=-28.
W(1,5)=-25.
W(1,6)=-20.
W(1,7)=-17.
W(1,8)=-14.
W(1,9)=-11.
W(1,10)=-9.
W(1,11)=-7.
W(1,12)=-5.
W(1,13)=-3.
W(1,14)=-2.
W(1,15)=-1.
W(1,16)=0.
W(2,1)=-43.
W(2,2)=-37.
W(2,3)=-33.
W(2,4)=-28.
W(2,5)=-25.
W(2,6)=-20.
W(2,7)=-17.
W(2,8)=-14.
W(2,9)=-11.
```

```
      W(2,10)=-9.
      W(2,11)=-7.
      W(2,12)=-5.
      W(2,13)=-3.
      W(2,14)=-2.
      W(2,15)=-1.
      W(2,16)=0.
      W(3,1)=-37.
      W(3,2)=-33.
      W(3,3)=-30.
      W(3,4)=-26.
      W(3,5)=-23.
      W(3,6)=-20.
      W(3,7)=-17.
      W(3,8)=-14.
      W(3,9)=-11.
      W(3,10)=-9.
      W(3,11)=-7.
      W(3,12)=-5.
      W(3,13)=-3.
      W(3,14)=-2.
      W(3,15)=-1.
      W(3,16)=0.
      W(4,1)=-29.
      W(4,2)=-26.
      W(4,3)=-25.
      W(4,4)=-22.
      W(4,5)=-19.
      W(4,6)=-17.
      W(4,7)=-15.
      W(4,8)=-13.
      W(4,9)=-11.
      W(4,10)=-9.
      W(4,11)=-7.
      W(4,12)=-5.
      W(4,13)=-3.
      W(4,14)=-2.
      W(4,15)=-1.
      W(4,16)=0.
      W(5,1)=-18.
      W(5,2)=-16.
      W(5,3)=-16.
      W(5,4)=-13.
      W(5,5)=-12.
      W(5,6)=-10.
      W(5,7)=-9.
      W(5,8)=-8.
      W(5,9)=-7.
      W(5,10)=-6.
      W(5,11)=-5.
      W(5,12)=-4.
      W(5,13)=-3.
      W(5,14)=-2.
      W(5,15)=-1.
      W(5,16)=0.
      I=IFIX((DX+0.2)/0.1)+1
      J=IFIX((DY-0.1)/0.02)+1
      IF(J.LT.1)GOTO 1
      IF(I.LT.1)I=1
      IF(I.GT.5)I=5
      IF(J.GT.16)J=16
      DELX=DX-X(I)
      DELY=DY-Y(J)
      IF((I.EQ.5).OR.(J.EQ.16))GOTO 2
      DYD=DELX*(Z(I+1,J)-Z(I,J))+DELY*(Z(I,J+1)-Z(I,J))
      DYND=DELX*(W(I+1,J)-W(I,J))+DELY*(W(I,J+1)-W(I,J))
      YD=(Z(I,J)+DYD)*1.E-05
      YND=(W(I,J)+DYND)*1.E-05
      RETURN
1     YD=Z(3,1)*1.E-05
      YND=W(3,1)*1.E-05
      RETURN
```

```
2     YD=Z(I,J)*1.E-05
      YND=W(I,J)*1.E-05
      RETURN
      END
```

COMPUTER PROGRAM III

COST FUNCTION MINIMIZATION

THIS PROGRAM COMPREHENDS

A) A MAIN (CALLING) PROGRAM

B) SUBROUTINE BOXPLX (FUNCTION MINIMIZATION)

C) FUNCTION FE (EVALUATION OF THE COST FUNCTION)

D) FUNCTION KE (IMPLICIT CONSTRAINTS)

E) FUNCTION RKLDEQ (INTEGRATION OF SIMULTANEOUS FIRST ORDER DIFFERENTIAL EQUATION USING THE FOURTH-ORDER RUNGE-KUTTA METHOD)

F) SUBROUTINE FORCES (NON-DIMENSIONAL VALUES OF FORCES AND MOMENTS)

DIMENSION X(8),XS(8),BU(8),BL(8)

INPUT DATA FOR BOXPLX

BL=UPPER BOUNDS FOR THE VARIABLES

BU=LOWER BOUNDS FOR THE VARIABLES

XS=STARTING VALUES OF THE VARIABLES

NT=ALLOWED NUMBER OF TRIALS

```
      READ(5,1)(BU(I),I=1,8)
      READ(5,1)(XS(I),I=1,8)
      READ(5,1)(BL(I),I=1,8)
      READ (6,4)NT
      WRITE(6,2)(BL(I),I=1,8)
      WRITE(6,2)(XS(I),I=1,8)
      WRITE(6,2)(BU(I),I=1,8)
      WRITE(6,4)NT
      CALL BOXPLX(8,0,0,NT,0.,XS,BU,BL,X,Y,N,M)
      WRITE(6,2)(X(I),I=1,8)
      WRITE(6,3)Y,N,M
1     FORMAT(8F10.5)
2     FORMAT(' ',8F10.5)
3     FORMAT(' ',F10.5,5X,2I10)
4     FORMAT(1I4)
      STOP
      END
```

```
      SUBROUTINE BOXPLX   (NV,NAV,NPR,NTZ,RZ,XS,BU,BL,XMN,YMN,NMN,MMN)    LIM03980
      DIMENSION V(50,50), FUN(50), SUM(25), CEN(25), XS(25), BU(25),     LIM04000
     1    BL(25), XMN(25), U(2500)                                       LIM04010
      EQUIVALENCE (V,U)                                                  LIM04020
C                                                                        LIM04030
C  IF KKON=0, EXPLICIT CONSTRAINTS ARE NOT APPLIED AFTER INITIAL         LIM04040
C              COMPLEX  IS FORMED                                        LIM04050
C                                                                        LIM04060
      KKON = 1                                                           LIM04070
      IXR=50                                                             LIM04080
      EP=1.E-7                                                           LIM04090
      IF(NTZ) 1799,1799,1798                                             LIM04100
 1799 NTA=2000                                                           LIM04110
      GO TO 1797                                                         LIM04120
 1798 NTA=NTZ                                                            LIM04130
 1797 R=RZ                                                               LIM04140
      IF(R) 1796,1796,1795                                               LIM04150
 1795 IF(R-1.)1794,1796,1796                                             LIM04160
 1796 R=1./3.                                                            LIM04170
 1794 CONTINUE                                                           LIM04180
      NVT=NV+NAV                                                         LIM04190
C           TOTAL VARS, EXPLICIT PLUS IMPLICIT                           LIM04200
      NT=0                                                               LIM04210
C           CURRENT TRIAL NO.                                            LIM04220
      NPT=0                                                              LIM04230
C           CURRENT NO. OF PERMISSIBLE TRIALS                            LIM04240
      NTFS=0                                                             LIM04250
C           CURRENT NO. OF TIMES F HAS BEEN ALMOST UNCHANGED             LIM04260
C                                                                        LIM04270
C             CHECK FEASIBILITY OF START POINT                           LIM04280
      DO 100 I=1,NV                                                      LIM04290
      BL(I) = BL(I) + AMAX1 (EP, EP*ABS(BL(I)))                          LIM04300
      BU(I) = BU(I) - AMAX1 (EP, EP*ABS(BU(I)))                          LIM04310
      VT=XS(I)                                                           LIM04320
      IF(BL(I)-VT)102,102,108                                            LIM04330
  108 II = -I                                                            LIM04340
      VT = BL(I)                                                         LIM04350
      GO TO 101                                                          LIM04360
  102 IF( BU(I) - VT) 110, 109, 109                                      LIM04370
  110 II = I                                                             LIM04380
      VT = BU(I)                                                         LIM04390
  101 WRITE (6,104) II                                                   LIM04400
  104 FORMAT(50H0INDEX AND DIRECTION OF OUTLYING VARIABLE AT START I5)   LIM04410
  109 V(I,1) = VT                                                        LIM04420
      CEN(I) = VT                                                        LIM04430
      BL(I) = BL(I) + AMAX1(EP, EP * ABS ( BL(I)))                       LIM04440
      BU(I) = BU(I) - AMAX1(EP   ,EP    *ABS (BU(I)))                    LIM04450
  100 SUM(I)=VT                                                          LIM04460
```

```
C
      NCE=1
C              NUMBER OF CONSTRAINT EVALUATIONS
      I=1
      IF (KE(V)) 162,106,162
  162 IF (NPR) 161, 161, 163
  163 WRITE (6,164)
  164 FORMAT (50HOIMPLICIT CONSTRAINT VIOLATED AT START. DEAD END.   )
      GO TO 161
  106 NFE=1
C              NUMBER OF FUNCTION EVALUATIONS
      NLIM = 5*NV +100
C              NO. OF HALF-WAY DISPLACEMENTS ALLOWED PER TRIAL
      BESTFU =1.E70
      K=2*NV
      ALPHA=1.3
      FK=K
      FKM=FK-1.
      BETA=ALPHA+1.
      IQR = R * 1.E7
      IF (MOD(IQR,2).EQ.0) IQR=IQR+101
C
C                        SET UP INITIAL VERTICES
      FUN(1) = FE(V)
  103 FI=1.
      FUNOLD=FUN(1)
      DO 150 I=2,K
      FI=FI+1.
      DO 151 J=1,NV
      CALL RANDU (IQR,IQR,RQX)
  151 V(J,I)=BL(J)+RQX       *(BU(J)-BL(J))
C
      DO 152 L=1,NLIM
      NCE=NCE+1
      IF (KE(V(1,I)))153, 157, 153
  153 DO 154 J=1,NV
  154 V(J,I)=.5*(V(J,I)+CEN(J))
  152 CONTINUE
      IF (NPR) 161,161,158
  158 WRITE (6,155) I
  155 FORMAT(22HOCANNOT FIND FEASIBLE  I4,19HTH VERTEX AT START    )
      CALL BOUT(NT ,NPT,NFE,NCE,NV,NVT,V,I,FUN,CEN )
  161 NMN = -I
      GO TO 1055
  157 DO 156 J=1,NV
      SUM(J)=SUM(J)+V(J,I)
  156 CEN(J)=SUM(J)/FI
      NFE=NFE+1
```

```
      FUN(I) = FE(V(1,I))
  150 CONTINUE
C
      IF (NPR) 159,159,160
  160 CALL BOUT (NT,NPT,NFE,NCE,NV,NVT,V,K,FUN,CEN)
C
  159 FX = FNM (FUN, 0, K, MN)
      M = MN
C
C        BASIC LOOP, ELIMINATE EACH WORST VERTEX IN TURN
C
  208 FUNMAX = FNM(FUN, M, K, NM)
C              M  IS INDEX OF WORST VERTEX AND NEXT WORST IS VERTEX NM,
C                 THE ACTUAL VALUE OF WHICH IS IN FUNMAX.
      LIMT = 5*NV         +100
      J=(M-1)*IXR
      J1=J+1
      DO 202 I=1,NV
      IJ=J+I
      VT=U(IJ)
      SUM(I) = SUM(I) - VT
      CEN(I) = SUM(I)/FKM
  202 U(IJ)=BETA*CEN(I)-ALPHA*VT
      NT=NT+1
C              TEST FIRST FOR CONSTRAINT VIOLATION
  213 IF(KKCN) 2131,2131,2132
 2132 DO 214 I=1,NV
      IJ=J+I
      VT=AMIN1(U(IJ),BU(I))
  214 U(IJ)=AMAX1(VT,BL(I))
 2131 DO 210 N=1,NLIM
      NCE=NCE+1
      IF(KE(U( J1)))207, 204, 207
  207 DO 209 I=1,NV
      IJ=J+I
  209 U(IJ)=.5*(CEN(I)+U(IJ))
      NT=NT+1
  210 CONTINUE
 2100 IF(NPR) 2102,2102,2101
 2101 WRITE (6,221) NT, M
  221 FORMAT (10HOAT TRIAL   I4,29H CANNOT FIND FEASIBLE VERTEX  I4 ,
     1  15X,   21HRESTART FROM CENTROID   )
      CALL BOUT (NT,NPT,NFE,NCE,NV,NVT,V,K,FUN,CEN)
 2102 DO 2210 I = 1, NV
      SUM(I)=CEN(I)
 2210 V(I,1) = CEN(I)
      KQX=KE(V(1,1))
      NCE=NCE+1
```

```
      NFE=NFE+1
      FUN(1) = FE(U(J1))
      TFUN=FUN(1)
      IF (TFUN - BESTFU ) 2212, 2212, 2218
 2218 IF(NPR) 1051, 1051, 2213
 2213 WRITE (6,2214)
 2214 FORMAT (27HOPREVIOUS MINIMUM WAS BEST.   )
      GO TO 1051
 2212 IF(NPR) 2215, 2215, 2216
 2216 WRITE (6,303) TFUN
 2215 YMN = TFUN
      DO 2217 IL = 1, NVT
 2217 XMN(IL) = CEN(IL)
      ABFUN=ABS (TFUN-BESTFU )
      CRIFUN=AMAX1(ABS (TFUN)*EP,EP)
      IF (ABFUN - CRIFUN) 1051,1051,2211
 2211 BESTFU =TFUN
      GO TO 103
C
  204 NFE=NFE+1
      FUNTRY=FE(V(1,M))
C
      IF(ABS (FUNTRY-FUNOLD)-AMAX1(ABS (EP    *FUNOLD),EP     ))217,217,
     1  218
  217 NTFS=NTFS+1
      IF(NTFS-K)211,300,300
  218 NTFS=0
  211 IF(FUNTRY-FUNMAX)212,212,215
  215 DO 216 I=1,NV
      IJ=J+I
  216 U(IJ)=.5*(CEN(I)+U(IJ))
      LIMT=LIMT-1
      IF(LIMT) 2100,2160,2160
 2160 NT=NT+1
      GO TO 213
C
  212 FUN(M)=FUNTRY
      FUNOLD = FUNTRY
      NPT=NPT+1
      DO 203 I=1,NV
      IJ=J+I
  203 SUM(I)=SUM(I)+U(IJ)
C
      M = NM
      IF (NPR) 400,400,401
  401 IF( MOD (NPT,NPR)) 208,205,208
  205 IYS = 1
  301 DO 222 IL = 1,NV
```

```
  222 CEN(IL) = SUM(IL)/FK                                                  LIM05910
      CALL BOUT(NT ,NPT,NFE,NCE,NV,NVT,V,K,FUN,CEN )                        LIM05920
      GO TO (402,403), IYS                                                  LIM05930
  402 IF (NT - NTA) 208,219,219                                             LIM05940
  219 WRITE (6,220)                                                         LIM05950
  220 FORMAT (27HOLIMIT ON TRIALS EXCEEDED.   )                             LIM05960
  403 TFUN = FE(CEN)                                                        LIM05970
      NFE=NFE+1                                                             LIM05980
      WRITE (6,303) TFUN                                                    LIM05990
  303 FORMAT(8HOMINIMUM,E20.7)                                              LIM06000
      GO TO 1050                                                            LIM06010
  400 IF (NT - NTA) 208,105,105                                             LIM06020
  105 DO 1053 IL = 1,NV                                                     LIM06030
 1053 CEN(IL) = SUM(IL)/FK                                                  LIM06040
      NFE=NFE+1                                                             LIM06050
      TFUN = FE(CEN)                                                        LIM06060
 1050 YMN = TFUN                                                            LIM06070
      DO 1052 I=1,NV                                                        LIM06080
 1052 XMN(I)= CEN(I)                                                        LIM06090
 1051 CALL KE(XMN)                                                          LIM06100
      NMN = NT                                                              LIM06110
      MMN = NPT                                                             LIM06120
 1055 RETURN                                                                LIM06130
  300 FK = FKM                                                              LIM06140
      IF (NPR) 105,105,404                                                  LIM06150
  404 WRITE (6,302) K                                                       LIM06160
  302 FORMAT (40HOFUNCTION HAS BEEN ALMOST UNCHANGED FOR I5, 7H TRIALS)     LIM06170
      IYS = 2                                                               LIM06180
      GO TO 301                                                             LIM06190
      END                                                                   LIM06200
C   FIND NEXT TO MAXIMUM VERTEX                                             LIM06210
      FUNCTION FNM (FUN, M, K, NM)                                          LIM06220
      DIMENSION FUN(50)                                                     LIM06230
      FU = -1.E70                                                           LIM06240
      DO 3 I = 1,K                                                          LIM06250
      IF (I-M) 2, 3, 2                                                      LIM06260
    2 IF (FU - FUN(I)) 1,3,3                                                LIM06270
    1 FU = FUN(I)                                                           LIM06280
      NM = I                                                                LIM06290
    3 CONTINUE                                                              LIM06300
      FNM = FU                                                              LIM06310
      RETURN                                                                LIM06320
      END                                                                   LIM06330
      SUBROUTINE RANDU (IX,IY,YFL)                                          LIM06340
      IY=IX*65539                                                           LIM06350
      IF(IY) 5,6,6                                                          LIM06360
    5 IY = IY + 2147483647 + 1                                              LIM06370
    6 YFL = IY                                                              LIM06380
```

```
      YFL = YFL * .4656613E-9
      RETURN
       END
C
      SUBROUTINE BOUT (NT,NPT,NFE,NCE,NV,NVT,V,K,FN,C)
      DIMENSION V(50,50),FN(50),C(25)
      WRITE (6,1) NT,NPT,NFE,NCE
    1 FORMAT (18HONUMBER OF TRIALS  I4,5X,20H PERMISSIBLE TRIALS  I4,
     1 5X, 22H FUNCTION EVALUATIONS  I4,5X,24H CONSTRAINT EVALUATIONS
     2 I4/ 1H0,25X, 21H- - - VERTICES - - -    )
C
      DO 4 I=1,K
      WRITE (6,2)  FN(I), (V(J,I), J=1,NV)
    2 FORMAT (1H , E18.7 ,2X,7E14.7 /(21X, 7E14.7))
      NVP=NV+1
    4 WRITE (6,3) (V(J,I),J=NVP,NVT)
    3 FORMAT (21X,7E14.7)
C
      WRITE (6,5) (C(I), I=1, NV)
    5 FORMAT (10HOCENTROID  11X, 7E14.7/(21X,7E14.7))
      RETURN
      END
```

```
      FUNCTION FE(Z)

      EVALUATION OF THE COST FUNCTION FOR A SET OF VALUES
      OF THE FEEDBACK LOOP GAINS, GENERATED BY BOXPLX

      IMPLICIT REAL * 8 (A-E,G-I,K-Y)
      DIMENSION Z(8),Y(20),YDOT(20)

      HYDRODYNAMIC COEFFICIENTS

      A11=0.015
      B11=0.01243
      A21=0.00027
      B21=0.0051
      A12=0.000197
      B12=0.00351
      A22=0.00068
      B22=0.00227
      A33=0.0085
      B33=0.0012
      KA=0.0027
      KB=-0.00126
      NC=0.0012
      D=A11*A22-A21*A12

      IDENTIFICATION OF THE VARIABLES

      K2=DBLE(Z(1))
      KT2=DBLE(Z(2))
      K1=DBLE(Z(3))
      KT1=DBLE(Z(4))
      KY2=DBLE(Z(5))
      KTY2=DBLE(Z(6))
      KY1=DBLE(Z(7))
      KTY1=DBLE(Z(8))
      KYI=0.25074D-02
      KTYI=0.63325D-01

      INITIAL CONDITIONS

      DO 1 J=1,20
    1 Y(J)=0.

      INITIAL LATERAL SEPARATION BETWEEN THE SHIPS

      Y(10)=0.2
      Y(15)=Y(10)

      Y(16)=Y(10)-Y(5)

      DESIRED FINAL SEPARATION BETWEEN THE SHIPS

      DFIN=0.10
```

```
      T=0.
      JT=0
      DT=0.3
      DYDOT1=0.
      DYDOT2=0.
      DYDOTI=0.0
3     DX=Y(18)-Y(17)
      ZX=SNGL(DX)
      ZY=SNGL(Y(16))
      CALL FORCES(ZX,ZY,ZD,ZND)
      YI=DBLE(ZD)
      NI=DBLE(ZND)

      DDC=COURSE CONTROL LOOP ACTION

      DDD=DISTANCE CONTROL LOOP ACTION

      DDC1=K1*Y(2)+KT1*Y(3)
      DDD1=KY1*(DFIN-Y(16))+KTY1*(DYDOT1-DYDOT2)
      DDD2=KY2*(Y(16)-DFIN)+KTY2*(DYDOT2-DYDOT1)
      DDC2=K2*Y(7)+KT2*Y(8)

      DD1=DDC1+DDD1
      DD2=DDC2+DDD2
      DDI=KYI*(Y(15)-DFIN)+KTYI*DYDOTI

      II11=KA*DD1+YI
      II12=KA*DD2-YI
      II1I=KA*DDI
      II21=KB*DD1+NI
      II22=KB*DD2-NI
      II2I=KB*DDI
      II31=NC
      II32=NC
      II3I=NC
      I11=-B11*Y(1)-B21*Y(3)+II11
      I12=-B11*Y(6)-B21*Y(8)+II12
      I1I=-B11*Y(11)-B21*Y(13)+II1I
      I21=-B12*Y(1)-B22*Y(3)+II21
      I22=-B12*Y(6)-B22*Y(8)+II22
      I2I=-B12*Y(11)-B22*Y(13)+II2I
      I31=-B33*Y(4)+II31
      I32=-B33*Y(9)+II32
      I3I=-B33*Y(14)+II3I

      YDOT(1)=(A22*I11-A21*I21)/D
      YDOT(2)=Y(3)
      YDOT(3)=(A11*I21-A12*I11)/D
      YDOT(4)=I31/A33
      YDOT(5)=Y(4)*DSIN(Y(2))+Y(1)*DCOS(Y(2))
      YDOT(6)=(A22*I12-A21*I22)/D
      YDOT(7)=Y(8)
      YDOT(8)=(A11*I22-A12*I12)/D
      YDOT(9)=I32/A33
      YDOT(10)=Y(9)*DSIN(Y(7))+Y(6)*DCOS(Y(7))
      YDOT(11)=(A22*I1I-A21*I2I)/D
      YDOT(12)=Y(13)
      YDOT(13)=(A11*I2I-A12*I1I)/D
      YDOT(14)=I3I/A33
      YDOT(15)=Y(14)*DSIN(Y(12))+Y(11)*DCOS(Y(12))
      YDOT(16)=YDOT(10)-YDOT(5)
      YDOT(17)=Y(4)*DCOS(Y(2))-Y(1)*DSIN(Y(2))
      YDOT(18)=Y(9)*DCOS(Y(7))-Y(6)*DSIN(Y(7))
      YDOT(19)=Y(14)*DCOS(Y(12))-Y(11)*DSIN(Y(12))
```

```
      EVALUATION OF THE COST FUNCTION

      YDOT(20)=10.*Y(5)**2+(Y(10)-Y(15))**2

      ZS=RKLDEQ(20,Y,YDOT,T,DT,JT)
      IF(ZS-1.)4,3,2
4     WRITE(6,5)
5     FORMAT(' ','TROUBLESOME INTEGRATION')
      STOP
2     DYDOT1=YDOT(5)
      DYDOT2=YDOT(10)
      DYDOTI=YDOT(15)
      IF(T-60.)3,3,7
7     FE=SNGL(Y(20))
      RETURN
      END


      FUNCTION KE(X)


      EVALUATION OF IMPLICIT CONSTRAINTS

      DIMENSION X(8)
      KE=0
      RETURN
      END
```

```
      FUNCTION RKLDEQ (N,Y,F,X,H,NT)


      REAL*8 Y,F,X,H,Q,H1,H2,H3,H6
      DIMENSION Y(1), F(1), Q(25)
      NT = NT +1
      GO TO (1,2,3,4),NT
    1 H1 = H
      H2 = H1 * .5D0
      H3 = H1 * 2.D0
      H6 = H1/6.D0
      DO 11 J =1,N
   11 Q(J) = 0.D0
      A = .5D0
      X = X + H2
      GO TO 5
    2 A = .29289321881345254
      GO TO 5
    3 A = 1.7071067811865475
      X = X + H2
      GO TO 5
    4 DO 41 I = 1,N
   41 Y(I) = Y(I) + H6 * F(I) -Q(I)/3.D0
      NT = 0
      RKLDEQ =2.
      GO TO 6
    5 DO 51 L = 1,N
      Y(L) = Y(L) + A *(H * F(L) -Q(L))
   51 Q(L) = H3 * A *F(L) +(1.D0-3.D0*A) *Q(L)
      RKLDEQ =1.
    6 RETURN
      END
```

COMPUTER PROGRAM IV

```
*       CONTROLLED SYSTEM RESPONSE

INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=2

*       HYDRODYNAMIC COEFFICIENTS

PARAM MXUD=-0.0085,XU=-0.0012
PARAM MYR=-0.0051,YRD=-0.00027
PARAM YV=-0.01243,MYVD=-0.015
PARAM NVD=-0.000197,NV=-0.00351
PARAM NR=-0.00227,IZNRD=-0.00068
PARAM YDEL=0.0027,NDEL=-0.00126

*       INITIAL DISTANCES FROM EACH SHIP TO THE AXES

INCON Y10=0.,Y20=0.20
INCON X10=0.,X20=0.

PARAM YI=0.,NI=0.

*       DESIRED FINAL SEPARATION BETWEEN THE SHIPS

PARAM DFIN=0.10

*       OPTIMAL FEEDBACK LOOP GAINS

PARAM K2=2.149,KT2=2.558,K1=3.814,KT1=2.810
PARAM KY2=0.5497,KTY2=2.4381,KY1=0.00244,KY1=2.953,KTY1=2.04

INITIAL

*       CALCULATION OF THE COEFFICIENTS

        A11=-MYVD
        B11=-YV
        C11=0.
        A21=-YRD
        B21=-MYR
        C21=0.
        A12=-NVD
        B12=-NV
        C12=0.
        A22=-IZNRD
        B22=-NR
        C22=0.
        A33=-MXUD
        B33=-XU
        NC=-XU
        KA=YDEL
        KB=NDEL
        D=A11*A22-A12*A21
```

```
*       INITIAL LATERAL SEPARATION BETWEEN THE SHIPS

        DY=Y20-Y10
        DX=X20-X10

        CALL FORCES(DX,DY,YI,NI)

*       SIMULATION

DERIVATIVE
        YDOT1=CDOT1*SIN(B1)+ADOT1*COS(B1)
        YDOT2=CDOT2*SIN(B2)+ADOT2*COS(B2)
        XDOT1=CDOT1*COS(B1)-ADOT1*SIN(B1)
        XDOT2=CDOT2*COS(B2)-ADOT2*SIN(B2)
        ADD1=(A22*I11-A21*I21)/D
        ADD2=(A22*I12-A21*I22)/D
        BDD1=(A11*I21-A12*I11)/D
        BDD2=(A11*I22-A12*I12)/D
        CDD1=I31/A33
        CDD2=I32/A33
        ADOT1=INTGRL(0.,ADD1)
        ADOT2=INTGRL(0.,ADD2)
        BDOT1=INTGRL(0.,BDD1)
        BDOT2=INTGRL(0.,BDD2)
        CDOT1=INTGRL(0.,CDD1)
        CDOT2=INTGRL(0.,CDD2)
        A1=INTGRL(0.,ADOT1)
        A2=INTGRL(0.,ADOT2)
        B1=INTGRL(0.,BDOT1)
        B2=INTGRL(0.,BDOT2)
        C1=INTGRL(0.,CDOT1)
        C2=INTGRL(0.,CDOT2)
        Y1=INTGRL(Y10,YDOT1)
        Y2=INTGRL(Y20,YDOT2)
        X1=INTGRL(X10,XDOT1)
        X2=INTGRL(X20,XDOT2)
        I11=-B11*ADOT1-C11*A1-B21*BDOT1-C21*B1+IF11
        I12=-B11*ADOT2-C11*A2-B21*BDOT2-C21*B2+IF12
        I21=-B12*ADOT1-C12*A1-B22*BDOT1-C22*B1+IF21
        I22=-B12*ADOT2-C12*A2-B22*BDOT2-C22*B2+IF22
        I31=-B33*CDOT1+IF31
        I32=-B33*CDOT2+IF32
        AF11=REALPL(0.,0.1,KA*DD1)
        AF12=REALPL(0.,0.1,KA*DD2)
        AF21=REALPL(0.,0.1,KB*DD1)
        AF22=REALPL(0.,0.1,KB*DD2)
        IF11=AF11+YI
        IF12=AF12-YI
        IF21=AF21+NI
        IF22=AF22-NI
        IF31=NC
        IF32=NC
        DYDOT=YDOT2-YDOT1

*       DDC=COURSE CONTROL ACTION

*       DDD=DISTANCE CONTROL ACTION

        DDC1=K1*B1+KT1*BDOT1
        DDD1=KY1*(DFIN-DY)-KTY1*DYDOT
        DDC2=K2*B2+KT2*BDOT2
        DDD2=KY2*(DY-DFIN)+KTY2*DYDOT
```

```
        CD1=DDC1+DDD1
        CD2=DDC2+DDD2

*       UP-DATED SEPARATION BETWEEN THE SHIPS

DYNAMIC
        DX=X2-X1
        DY=Y2-Y1

        CALL FORCES(DX,DY,YI,NI)
SAMPLE
PRINT 0.8,DX,DY,Y1,Y2,B1,B2
PREPAR 0.4,Y1,Y2,B1,B2,DX,DY,X1,X2
CONTRL FINTIM=40.,DELT=0.04,DELS=0.1
GRAPH SAME,TIME,Y1,Y2
GRAPH SAME,TIME,B1,B2
GRAPH TIME,DY,DX
GRAPH SAME,X1,Y1,Y2
PRPLOT ONLY
        IF(DY.LE.0.05)WRITE(6,3)
    3   FORMAT(' ','LATERAL SEPARATION LESS THAN 25 FT')
        CALL DRWG(1,1,TIME,Y1)
        CALL DRWG(1,2,TIME,Y2)
        CALL DRWG(2,1,TIME,B1)
        CALL DRWG(2,2,TIME,B2)
        CALL DRWG(3,1,TIME,DY)
        CALL DRWG(3,2,TIME,DX)
        CALL DRWG(4,1,X1,Y1)
        CALL DRWG(4,2,X1,Y2)
TERMINAL
        CALL ENDRW(NPLOT)
END
INCON Y10=0.,Y20=0.4
PARAM DFIN=0.24
PARAM K2=2.917,KT2=2.042,K1=2.975,KT1=2.845
PARAM KY2=1.506,KTY2=2.812,KY1=2.975,KTY1=2.727
END

INCON Y10=0.0,Y20=0.36
PARAM DFIN=0.20
PARAM K2=2.853,KT2=2.141,K1=3.069,KT1=3.080
PARAM KY2=0.533,KTY2=2.208,KY1=3.520,KTY1=2.555
END
STOP
```

```
//LIMA$ID2 JOB (0709,0500,EA24),'LIMA',TIME=4,MSGLEVEL=(0,0)
// EXEC DSL
//DSL.INPUT DD *


*       COMPUTER PROGRAM V

*       IDEALIZED RESPONSE FOR THE TRACKING SHIP

INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=1

*       HYDRODYNAMIC COEFFICIENTS

CONST NR=-0.00227,NV=-0.00351,NVD=-0.000197
CONST MYVD=0.015,MYR=0.0051,IZNRD=0.00068,MXUD=0.0085
CONST YV=-0.01243,XU=-0.0012,YRD=-0.00027
CONST YDELR=-0.0027,NDELR=-0.00126,XDELR=0.0

*       DESIRED FINAL DISTANCE TO THE X-AXIS

        DFIN=0.1

*       INITIAL DISTANCES TO THE AXES

INCON X0=0.,Y0=0.2

*       FEEDBACK LOOP GAINS FOR CRITICALLY DAMPED RESPONSE

PARAM K=0.25074E-02,KT=0.63325E-01

*       CALCULATION OF THE COEFFICIENTS

INITIAL
        A11=MYVD
        B11=-YV
        A21=-YRD
        B21=MYR
        A12=-NVD
        B12=-NV
        A22=IZNRD
        B22=-NR
        A33=MXUD
        B33=-XU
        NC=-XU
        KA1=-YDELR
        KB1=NDELR
        KC1=XDELR
        C=A11*A22-A12*A21

DERIVATIVE
        AF=K*(Y-DFIN)
        BF=KT*YDOT
        IF1=KA1*(AF+BF)
        IF2=KB1*(AF+BF)
        IF3=KC1*D1+NC
```

```
*       TIME DOMAIN SIMULATION

        I1=-B11*ADOT-B21*BDOT+IF1
        I2=-B12*ADOT-B22*BDOT+IF2
        I3=-B33*CDOT+IF3
        ADDOT=(I1*A22-I2*A21)/D
        BDDOT=(I2*A11-I1*A12)/D
        CDDOT=I3/A33
        ADOT=INTGRL(0.,ADDOT)
        BDOT=INTGRL(0.,BDDOT)
        CDOT=INTGRL(0.,CDDOT)
        A=INTGRL(0.,ADOT)
        B=INTGRL(0.,BDOT)
        C=INTGRL(0.,CDOT)
        XDOT=CDOT*COS(B)-ADOT*SIN(B)
        YDOT=CDOT*SIN(B)+ADOT*COS(B)
        X=INTGRL(XO,XDOT)
        Y=INTGRL(YO,YDOT)
*       PRINTED OUTPUT
SAMPLE
CONTRL  FINTIM=40.,DELT=0.1,DELS=0.2
PREPAR  0.2,Y,B
GRAPH  TIME,Y,B
PRPLOT  ONLY
        CALL  DRWG(1,1,TIME,Y)
TERMINAL
        CALL  ENDRW(NPLOT)
END
STOP

//PLOT.SYSIN DD *
```

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